

ST.MARY'S PUBLIC SCHOOL

Study Material



Note:-

1. Check the website regularly.
2. Visit relevant subject links.
3. Utilize your time well to explore, learn and share.

My dear students,

Hope you all are well. Please pay attention!

You are requested not to adjust with any short cut for your learning process. Before you start your assignment listen carefully to the links/ videos/ voice messages, we are uploading on the website as well as on the WhatsApp. If you have any doubt contact your teacher to get it cleared.

Week 3- Lesson and Assignments

FLAMINGO

L-3 DEEP WATER BY WILLIAM DOUGLAS

(<https://www.youtube.com/watch?v=sFg6SIT9wzE&feature=youtu.be>)

Answer the following –

Think as you read

Q. 1, 2 and 3 (page no. 27)

Q. 1,2 and 3 (page no.29)

Understanding the text

Q. 2 and 3 (page no.29)

Q. 1 (page no. 30)

Additional short answer questions:

1. Why was the YMCA pool considered safe? What did Douglas' Mother warn him about and why?
2. What was Douglas' first misadventure with water?
3. What did Douglas mean by saying "The instructor was finished, but I was not"?

POEM 3- KEEPING QUIET BY PABLO NERUDA

(<https://www.youtube.com/watch?v=tvVwcY2pe7w&feature=youtu.be>)

Short answer questions- Think it out

Q. 1,2,3 and 4 (page no. 96)

Reference to Context (refer Goyal's)

R.T.C. No. 1

“Perhaps the earth..... later proves to be alive.

Do all the 3 questions based on it.

R.T.C. No. 4

“Those who prepare green wars..... doing nothing.

Do all the 4 questions based on it.

R.T.C. No. 6

“It would be an exotic moment..... strangeness”

Do all the 4 questions based on it.

VISTAS

L- 3 JOURNEY TO THE END OF THE EARTH BY TISHANI DOSHI

(<https://www.youtube.com/watch?v=Rj9g0d3brJM&feature=youtu.be>)

Reading with insight

Q. 1,2,3and 4 (page no.23)

Additional questions

1. What are the reasons for the increasing global temperature?
2. What are the main features of the Antarctica region as discussed in the lesson?

Complete the assignments by the end of the week and keep it ready for checking.

All the Best. Stay Home Stay Safe.

CBSE Class 12 Business Studies
Revision Notes
CHAPTER – 3
BUSINESS ENVIRONMENT

Meaning of Business Environment:

Business environment refers to forces and institutions outside the firm with which its members must deal to achieve the organisational purposes. Here

- Forces = economical, social, political, technological etc
- Institutions = suppliers, customers, competitors etc

It includes all those constraints and forces external to a business within which it operates. therefore,

- The firm must be aware of these external forces and institutions and
- The firm must be nagged keeping in mind these forces and institutions so that the organisational objectives are achieved. .

Features of Business Environment

1. Totality of external forces: Business environment is the sum total of all the forces/factors external to a business firm.

2. Specific and general forces: Business environment includes both specific and general forces. Specific forces include investors, competitors, customers etc. who influence business firm directly while general forces include social, political, economic, legal and technological conditions which affect a business firm indirectly.

3. Inter-relatedness: All the forces/factors of a business environment are closely interrelated. For example, increased awareness of health care has raised the demand for organic food and roasted snacks.

4. Dynamic: Business environment is dynamic in nature which keeps on changing with the

change in technology, consumer's fashion and tastes etc.

5. Uncertainty: Business environment is uncertain as it is difficult to predict the future environmental changes and their impact with full accuracy.

6. Complexity: Business environment is complex which is easy to understand in parts separately but it is difficult to understand in totality.

7. Relativity: Business environment is a relative concept whose impact differs from country to country, region to region and firm to firm. For example, a shift of preference from soft drinks to juices will be welcomed as an opportunity by juice making companies while a threat to soft drink manufacturers.

IMPORTANCE OF BUSINESS ENVIRONMENT

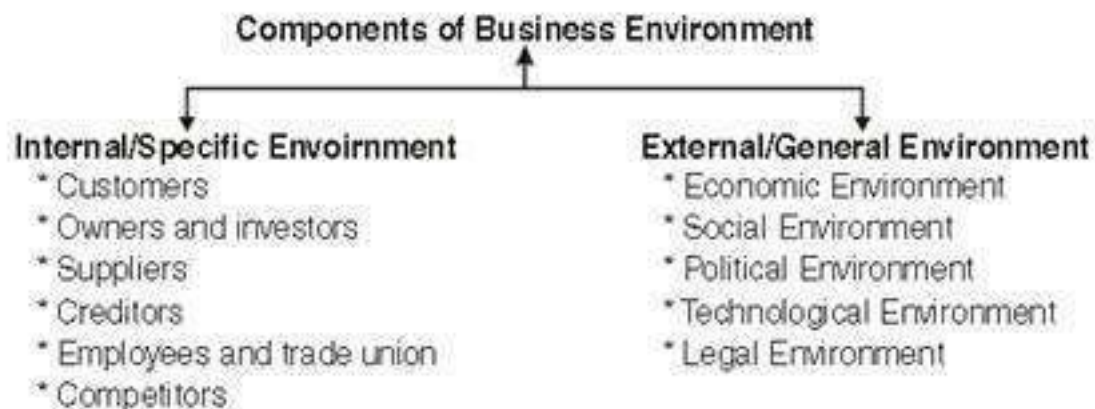
1. Identification of opportunities to get first mover advantage: Understanding of business environment helps an organization in identifying advantageous opportunities and getting their benefits prior to competitors, thus reaping the benefits of being a pioneer.

2. Identification of threats: Correct knowledge of business environment helps an organization to identify those threats which may adversely affect its operations. For example, Bajaj Auto made considerable improvements in its two wheelers when Honda & other companies entered the auto industry.

3. Tapping useful resources: Business environment makes available various resources such as capital, labour, machines, raw material etc. to a business firm. In order to know the availability of resources and making them available on time at economical price, knowledge of business environment is necessary.

4. Coping with Rapid changes: Continuous study/scanning of business environment helps in knowing the changes which are taking place and thus they can be faced effectively.

5. Assistance in planning and policy formulation: Understanding and analysis of business environment helps an organization in planning & policy formulation. For example, ITC Hotels planned new hotels in India after observing boom in tourism sector.



Helps in Improving performance: Correct analysis and continuous monitoring of business environment helps an organization in improving its performance.

Economic Environment in India

As a part of economic reforms, the Government of India announced New Economic Policy in July 1991 for taking out the country out of economic difficulty and speeding up the development of the country.

Main features of NEP, 1991 are as follows:

1. Only six industries were kept under licensing scheme.
2. The role of public sector was limited only to four industries.
3. Disinvestment was carried out in many public sector enterprises.
4. Foreign capital/investment policy was liberalized and in many sectors 100% direct foreign investment was allowed.
5. Automatic permission was given for signing technology agreements with foreign companies.
6. Foreign investment promotion board (FIPB) was setup to promote & bring foreign investment in India.
7. Various benefits were offered to small scale industries.

The three main strategies adopted for the above may be defined as follows:

1. Globalisation:

- Integrating the economy of a country with the economies of other countries to facilitate freer flow of trade, capital, persons and technology across borders. It leads to the emergence of a cohesive global economy.
- Till 1991, the Government of India had followed a policy of strictly regulating imports in value and volume terms. These regulations were with respect to (a) licensing of imports, (b) tariff restrictions and (c) quantitative restrictions.
- NEP '91 advocated rapid advancement in technology and directed trade liberalization towards:

a. Import Liberalisation

b. Export promotion towards rationalization of the tariff structure and

c. Reforms w.r.t foreign exchange

2. Liberalisation:

= Liberalising the Indian business and industry from all unnecessary controls and restrictions. That is relaxing rules and regulations which restrict the growth of the private sector and allowing the private sector to take part in economic activities that were earlier reserved for the government sector. The steps taken for this were:

- a. Abolishing licensing
- b. Freedom in deciding the scale of operations
- c. Removal of restrictions on movement of goods and services.
- d. Freedom in fixing prices.
- e. Reduction in tax rates and unnecessary controls
- f. Simplifying procedures for import and exports
- g. Making it easy to attract foreign capital.

3. Privatization:

- Refers to the reduction of the role of the public sector in the economy of a country.
- Transfer of ownership and control from the private to the public sector (disinvestment) can be done by : a. Sale of all/some assets of the public sector enterprises. b. Leasing of public enterprises to the private sector. c. Transfer of management of the public enterprise to the private sector.
- To achieve privatization in India, the government redefined the role of the public sector and -

a. Adopted a policy of planned disinvestment of the public sector

b. Refer the loss making and sick units to the Board of Industrial and Financial Reconstruction (BIFR)

DIMENSIONS/COMPONENTS OF BUSINESS ENVIRONMENT

1. Economic Environment: It has immediate and direct economic impact on a business. Rate of interest, inflation rate, change in the income of people, monetary policy, price level etc. are some economic factors which could affect business firms. Economic environment may offer opportunities to a firm or it may put constraints.

2. Social Environment: It includes various social forces such as customs, beliefs, literacy rate, educational levels, lifestyle, values etc. Changes in social environment affect an organization in the long run. Example: Now a days people are paying more attention towards their health, as a result of which demand for mineral water, diet coke etc. has increased while demand of tobacco, fatty food products has decreased.

3. Technological Environment: It provides new and advance ways/techniques of production. A businessman must closely monitor the technological changes taking place in the industry as it helps in facing competition and improving quality of the product. For Example, Digital watches in place of traditional watches, artificial fabrics in place of traditional cotton and silk fabrics, booking of railway tickets on internet etc.

4. Political Environment: Changes in political situation also affect business organizations. Political stability builds confidence among business community while political instability and bad law & order situation may bring uncertainty in business activities. Ideology of the political party, attitude of government towards business, type of government-single party or coalition government affects the business Example: Bangalore and Hyderabad have become the most popular locations for IT due to supportive political climate.

5. Legal Environment: It constitutes the laws and legislations passed by the Government, administrative orders, court judgements, decisions of various commissions and agencies. Businessmen have to act according to various legislations and their knowledge is very necessary. Example: Advertisement of Alcoholic products is prohibited and it is compulsory to give statutory warning on advertisement of cigarettes.

MAJOR STEPS IN ECONOMIC FORMS

1. New Industrial Policy - Under this the industries have been freed to a large extent from licences and other controls. Efforts have been made to encourage foreign investment.

2. New Trade Policy - The Foreign trade has been freed from the unnecessary control. The age old restrictions have been eliminated.

3. Fiscal Reforms. The greatest problem confronting the Indian Govt. is excessive fiscal deficit.

(a) Fiscal Deficit - It means country is spending more than its income

(b) Gross Domestic Product (GDP) - It is the sum total of the financial value of all goods & services produced in a year in a country.

4. Monetary Reform - It is a sort of control policy through which the central bank controls the supply of money with a view to achieving objectives of general economic policy.

5. Capital Market Reforms- The Govt. has taken the following steps for the development of this market:

(1) SEBI has been established.

(2) The restriction in respect of interest on debentures has been lifted.

(3) Private Sector has been permitted to establish Mutual Fund.

6. Dismantling Price control - The govt. has taken steps to remove price control in many products especially in fertilizers, iron and steel, petro products. Restrictions on the import of these things have also been removed.

IMPACT OF GOVERNMENT POLICY CHANGES ON BUSINESS AND INDUSTRY

1. Increasing Competition: De-licencing and entry of foreign firms Indian market is increased the level of competition for Indian firms.

2. More Demanding Customers: Now customers are more aware and they keep maximum information of the market as the result of which now market is customer/buyer oriented, Now, products are produced keeping in mind the demands of the customers.

3. Rapid Changing Technological Environment: Rapid Technological advancement has changed/improved the production process as a result of which maximum production is possible at minimum cost but it leads to tough challenges in front of small firms.

4. Necessity for Change- After New Industrial. Policy the market forces (demand & supply) are changing at a very fast rate. Change in the various components of business environment has made it necessary for the business firms to modify their policies & operations from time to time.

5. Need for Developing Human Resources: The changing market conditions of today requires people with higher competence and greater commitment, hence there is a need for developing human resources which could increase their effectiveness and efficiency.

6. Market Orientation: Earlier selling concept was famous in the market now its place is taken by the marketing concept. Today firms produce those goods & services which are required by the customers. Marketing research, educational advertising, after sales services have become more significant.

7. Reduction in budgetary Support to Public Sector: The budgetary support given by the government to the public sector is reducing thus the public sector has to survive and grow by utilising their own resources efficiently.

Class XII

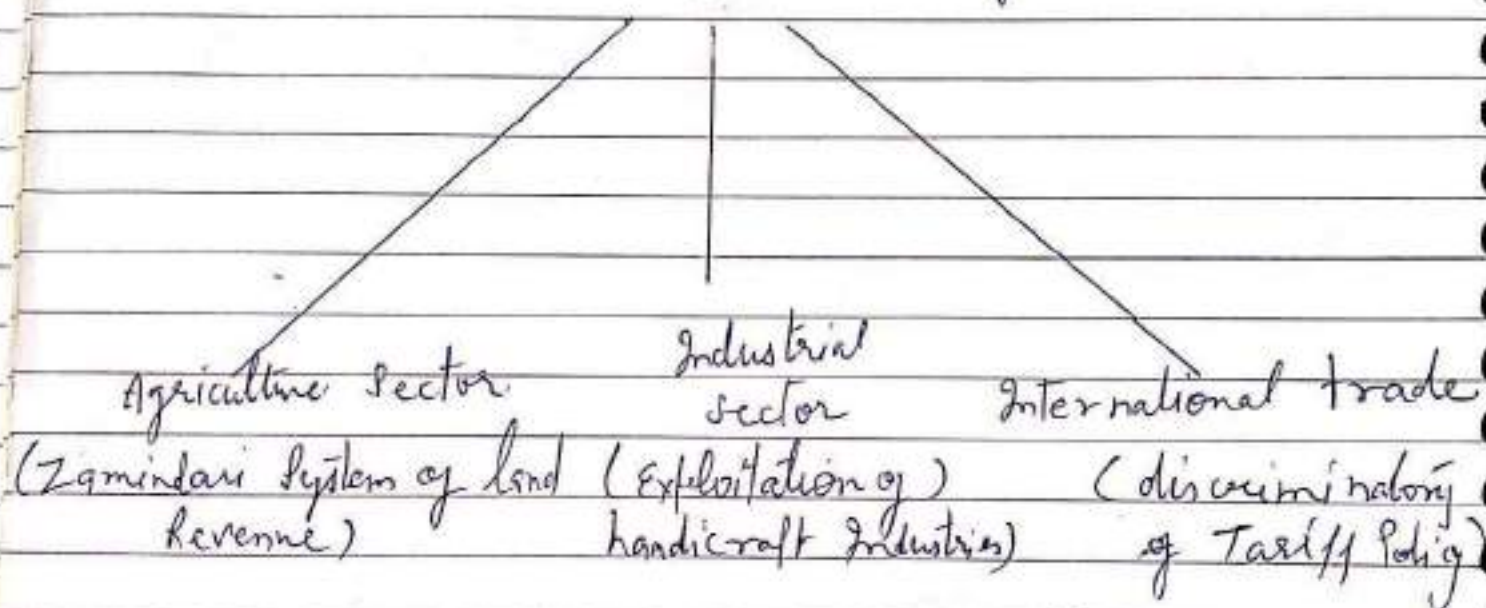
Indian Economic development

Chapter 1

Indian Economy on the eve of Independence

Colonial Exploitation of the Indian Economy under the British Rule

Colonial Exploitation of



1. Zamindars were free to collect as much as they wish revenue from the tillers.	1. Foreign demand for Indian handicraft was destroyed by heavy duty on their export.	1. Duty free export of Indian raw material to Britain.
2. Tillers were left with no surplus for investment.	2. Domestic demand by duty free import of British goods in the Indian market.	2. Duty free import of British goods to increase demand for British goods in the Indian market.
3. Zamindars spent revenue income on the luxuries of life.		

Features of Indian Economy on the eve of Independence

1. Stagnant Economy: Little or no growth in income
2. Backward Economy: \downarrow 1860-1950 PGI $\frac{0.5}{0.1}$ to $\frac{0.1}{0.1}$
Per capita income was just ₹ 230 in 1947-48
3. Agricultural Backwardness: -
engagement was 72% but contribution to GDP was only 50%.
Productivity was also low. wheat 660 Kg and Rice 665 Kg per hectare.
Foodgrain production was only 527 Lakh Tonnes, enough for subsistence.
4. Industrial Backwardness: -
Lack of basic and heavy industries, machine was almost negligible (dependent upon imports from Britain)
small scale and cottage industries were ruined
5. Rampant poverty: -
Lack of food, clothing and shelter
widespread poverty because of destruction of handicraft industries.
6. Poor Infrastructure: - means of communication, Transport, power generation were low. Power generation was only 2100 MW. Railway line was only 53,596 Km long.
7. Heavy dependence on imports: -
Our country is used to import armed forces, defence equipments, several consumer goods like sewing machines, medicines, kerosene oil, bicycle from (import) abroad.

8. Limited urbanisation — In 1948 only 14% population lived in urban areas while 86% in rural areas with no opportunities outside agriculture.

9. Semi-feudal Economy : — neither wholly feudal nor a capitalist. It was a mixed economy.

10. Colonial Economy : — heavy taxes by British on domestic industries.

Industries

free Export of raw material

Indian artisans were forced to close down their cottage industries.

Agriculture Sector on the eve of Independence

1. Production and productivity

Production means total output,

Productivity means output per hectare of land

Both were very low

2. High degree of uncertainty :-

No efforts were made under the British rule to develop permanent means of irrigation.

3. Dominance of subsistence farming

Subsistence farming

was only to provide the basic needs of the family, no surplus was left to sale in the market.

4. Gully between owners of the soil and tillers of the soil

owners were interested in maximising their rental income. The tillers of the soil were merely given enough for subsistence.

5. Small and fragmented holdings
landholdings were both small as well as a piece here and a piece there.

6. Land revenue system under the British Raj :
British Govt set up a triangular relationship among the Govt.

1. Zamindars were the permanent owner of land
2. They were to pay a fixed sum to the Govt
3. They were free to collect as much from the tillers as they could.

This led to unlimited exploitation of tillers, high rate of land revenue and became landless labourers.

Zamindars were having lavish lifestyle and spent all their income on luxuries of life. Improvement in agriculture was neglected.

Forced Commercialisation of Agriculture

Farmers were forced to produce cash crops (commercial crops) indigo in place of conventional subsistence crops.

Industrial sector on the eve of Independence

There was two fold motive behind the systematic de-industrialisation

systematic de-industrialisation

(1) Decay of world famous traditional handcraft industry due to discriminatory policies of Govt.

(2) Stagnant growth of industry due to lack of investment opportunities

1. Discriminatory Tariff policy of the state :-

1. Tariff free Export of raw material from India

2. Tariff free Import of British Industrial products

heavy duty on the export of Indian handicrafts

Indian handicrafts started losing domestic and foreign market

2. Disappearance of Princely courts :-

Prior to British rule, Nawabs, Rajas, Princes and emperors ruled different parts of the country. The beginning of British rule implied the end of princely courts.

3. Competition from machine-made products :-

machine-made products from Britain were low cost products and gave a stiff competition to the handicrafts products in India. Competition forced the Indian craftsmen to shut-down their enterprises.

4. New Patterns of demand :-

Due to the impact of British culture, a new class emerged in India which was keen to adopt the western lifestyle. This changed the pattern of demand in favour of the British products.

5. Introduction of Railways in India :-

With the introduction of railways, size of market for the British products tended to expand.

Bleak Growth of modern Industry

observations :-

① Industries established by Pvt. Ent.

Tata iron and Steel Industry (1907)
Sugar, Cement and Paper also

- (2) State participation was confined to Railways and means of Communications.
3. No Capital Industries.

Characteristics

1. Handicraft industry was destroyed by British Govt. using discriminatory policy.
2. Modern Industry was restricted to the Expansion of Railways.
3. No Capital goods industry.
4. While the traditional Indian industry were decaying, modern industry remained in an infant stage.

Foreign Trade under the British Rule :-

1. Net Exporter of Primary products and importer of finished goods :-
 owing to colonial exploitation of Indian Economy, India became net exporter of raw material and primary products like raw silk, cotton, wool, jute, indigo and sugar etc and importer of finished goods like cotton, silk and woollen clothes as well as capital goods also.
2. Monopoly Control of India's foreign trade :-
 More than 50% of India's trade with Great Britain.
 Exports and Imports both were under monopoly control of British Govt.

3. Surplus Trade but only to benefit the British :-

1. our surplus balance of trade was due to the export of primary products not the industrial goods, which is a sign of economic backwardness.

2. Trade Surplus was not used for Growth and development of Country. It was used to meet administrative expenses and war expenses. This led to a huge drain of wealth from India.

Demographic profile during the British rule :

1. Birth Rate and Death Rate :-

It was very high nearly 48 and 40 per thousand.

2. Infant mortality rate :-

It was about 218 per thousand. Presently it is 32 per thousand.

3. Life expectancy :-

It was 32 in contrast to 69.4 years presently.

4. Literacy rate

It was 16%, reflection of backwardness. female literacy rate was 7%.

Demographic Transition

Following are some notable points relating to demographic transition in India:

- (i) In the history of demographic transition, 1921 is regarded as the 'Year of Great Divide'.
- (ii) Prior to 1921, population growth in India was never consistent. Size of population kept fluctuating, increasing in one census and decreasing in the other.
- (iii) After 1921, population in India recorded a consistent rise.

7. OCCUPATIONAL STRUCTURE ON THE EVE OF INDEPENDENCE

Occupational structure refers to distribution of working population across primary, secondary and tertiary sectors of the economy.

Table 2 shows occupational structure of Indian economy at the time of independence. The data relates to the 1951, because reliable statistics for the year 1947 are not available.

Table 2. Occupational Distribution of India at the Time of Independence

Occupation	1951 (in %)
1. Primary Sector	72.7
(i) Agriculture	50.0
(ii) Agricultural Labour	19.7
(iii) Forestry, Fisheries, Animal Husbandry, Plantation	2.4
(iv) Mining	0.6
2. Secondary Sector	10.1
(v) Small and Large Scale Industries	9.0
(vi) Building Construction	1.1
3. Tertiary Sector	17.2
(vii) Trade and Commerce	5.2
(viii) Transport, Storage and Communication	1.4
(ix) Other Services	10.6
	100.00

[Source: Census of India 2011]

Table 2 offers the following observations:

- (1) **Agriculture—The Principal Source of Occupation:** On the eve of independence, about 72.7 per cent of working population was engaged in agriculture.

Percentage of population dependent on agriculture is much less in advanced countries of the world. For instance, in England and America 2 per cent, in Japan 12 per cent and in Germany 4 per cent of the population depend on agriculture.

This establishes backwardness of the Indian economy at the time of independence.

- (2) **Industry—An insignificant Source of Occupation:** On the eve of independence, barely 9.0 per cent of the working population in India was engaged in manufacturing industries, mining, etc.

As against it, 32 per cent in the USA, 42 per cent in England and 39 per cent in Japan are engaged in these activities.

It further proves how backward the Indian economy was at the time of independence.

- (3) **Unbalanced Growth:** The table shows unbalanced growth of the Indian economy.

Growth is said to be balanced when all sectors of the economy are equally developed. However, in case of India, secondary and tertiary sectors were in their infant stage of growth.

Hence, the conclusion that Indian economy at the time of independence was lopsided and therefore, backward.

Agriculture as a Means of Subsistence

- Greater dependence on agriculture (as suggested by occupational structure on the eve of independence) implied lesser availability of land per head of the farming population.
- Accordingly, agriculture was taken largely as a means of subsistence, and less as an occupation for profit.

- ❑ Assessed in terms of occupational distribution of the working population in India at the time of Independence, we get a disappointing picture of the Indian economy.
- ❑ Since bulk of the working population was engaged in agricultural sector (along with the fact that agriculture was merely a means of subsistence), Indian economy was in a state of extreme backwardness.
- ❑ The masses led their life in extreme poverty.

3. INFRASTRUCTURE ON THE EVE OF INDEPENDENCE

Infrastructure refers to the elements of (i) economic change (like means of transport, communication, banking, power/energy), and the elements of (ii) social change (like growth of educational, health and housing facilities), which serve as a foundation for growth and development of a country.

The state of India's infrastructure on the eve of independence can be described in terms of the following observations:

- (i) Railways were developed to transport finished goods from Britain to the interiors of the colonial India (with a view to widening the size of the market). It aimed at widening the size of the market for the British products in India.
- (ii) Ports were developed to handle export of raw material to Britain and import of finished goods from Britain.
- (iii) Post and telegraphs were developed to enhance administrative efficiency.
- (iv) Roads were developed to facilitate transportation of raw materials from different parts of the country to the ports.

Briefly, some modest infrastructural change in the economy during the British Raj is not denied. But, the motive behind this change was not the growth and development of the Indian economy; rather it was the growth and development of the British economy through colonial exploitation of the Indian economy. Consequently, Indian economy remained to be backward.

IMPACT OF RAILWAYS IN INDIA

Positive Impact

- (i) Railways facilitated expansion of the domestic market. Accordingly, exports and imports of the country showed a significant rise.
- (ii) Railways facilitated commercialisation of agriculture, as goods could then be moved to distant places. This implied a modest change in the outlook of the farmers. They started viewing farming as a business, rather than merely as a source of subsistence.
- (iii) Railways enabled people to break the barriers of distance and undertake journeys to far off places. This promoted cultural affinity among the countrymen.
- (iv) Faster movement of food grain across different parts of the country (owing to Railways) helped control the spread of famines. Food supplies could reach the people before they were driven to starvation.



Railways in the British Rule

Negative Impact

- (i) Railways contributed to colonial exploitation of the Indian economy. Because, primary goods (raw material) could then be easily transported from the fields and farms to the ports for the purpose of exports to the British economy.
- (ii) Finished goods coming as imports to the Indian economy could be easily transported to the interiors of the country for purpose of sale.
- (iii) Thus, the spread of railways led to the spread of the domestic market for the British products.

Was there any Positive Impact of the British Rule in India?

certainly not, if the impact of the British rule is studied with reference to the 'motive' of the British government in India. The motive was clear and focused: it was colonial exploitation of the Indian economy. However, the means to achieve the end yielded some positive side-effects. These are as under:

- (1) **Commercial Outlook of the Farmers:** Forced commercialisation of agriculture under the British rule exposed the subsistence farmers to uncertainties of the market. True, but it also led to a gradual change in outlook of the farmers. The farmers started considering market price of the produce as an important determinant of their production decisions.
- (2) **New Opportunities of Employment:** Spread of railways and roadways opened up new opportunities of economic and social growth.
- (3) **Control of Famines:** Rapid means of transport facilitated rapid movement of food grain to the famine-affected areas. Accordingly, famines were controlled.
- (4) **Monetary System of Exchange:** There was a transition from barter system of exchange to monetary system of exchange. Growth of monetary system of exchange facilitated division of labour, specialisation, and large-scale production.
- (5) **Efficient System of Administration:** The British government in India left a legacy of an efficient system of administration. This served as a ready-reference for our politicians and planners.

Computer Sci. & I. P.

Integrity Constraints

One of the major responsibility of a DBMS is to maintain the Integrity of the data i.e. Data being stored in the Database must be correct and valid.

An Integrity Constraints or Constraints are the rules, condition or checks applicable to a column or table which ensures the integrity or validity of data.

The following constraints are commonly used in MySQL.

- NOT NULL**
- PRIMARY KEY**
- UNIQUE ***
- DEFAULT ***
- CHECK ***
- FOREIGN KEY ***



Most of the constraints are applied with Column definition which are called **Column-Level (in-line Constraints)** ,but some of them may be applied at column Level as well as **Table-Level (Out-line constraints)** i.e. after defining all the columns. Ex.- Primary Key & Foreign Key



* Not included in the syllabus (recommended for advanced learning)

Type of Constraints

S.N	Constraints	Description
1	NOT NULL	Ensures that a column cannot have NULL value.
2	DEFAULT	Provides a default value for a column, when nothing is given.
3	UNIQUE	Ensures that all values in a column are different.
4	CHECK	Ensures that all values in a column satisfy certain condition.
5	PRIMARY KEY	Used to identify a row uniquely.
6	FOREIGN KEY	Used to ensure Referential Integrity of the data.

UNIQUE v/s PRIMARY KEY

- **UNIQUE** allows **NULL** values but **PRIMARY KEY** does not.
- Multiple column may have **UNIQUE** constraints, but there is only one **PRIMARY KEY** constraints in a table.

Implementing Primary Key Constraints

❖ Defining Primary Key at Column Level:

```
mysql> CREATE TABLE Student
  ( StCode  char(3)  NOT NULL PRIMARY KEY,
    Sname   char(20) NOT NULL,
    ..... *
  );
```

❖ Defining Primary Key at Table Level:

```
mysql> CREATE TABLE Student
  ( StCode  char(3)  NOT NULL,
    Sname   char(20) NOT NULL,
    ..... *
    PRIMARY KEY (StCode) );
```

Constraint is defined after all column definitions.

Implementing Constraints in the Table

```
mysql> CREATE TABLE Student
  (StCode char(3) NOT NULL PRIMARY KEY,
   Stname char(20) NOT NULL,
   StAdd varchar(40),
   AdmNo char(5) UNIQUE,
   StSex char(1) DEFAULT 'M',
   StAge integer CHECK (StAge>=5) );
```

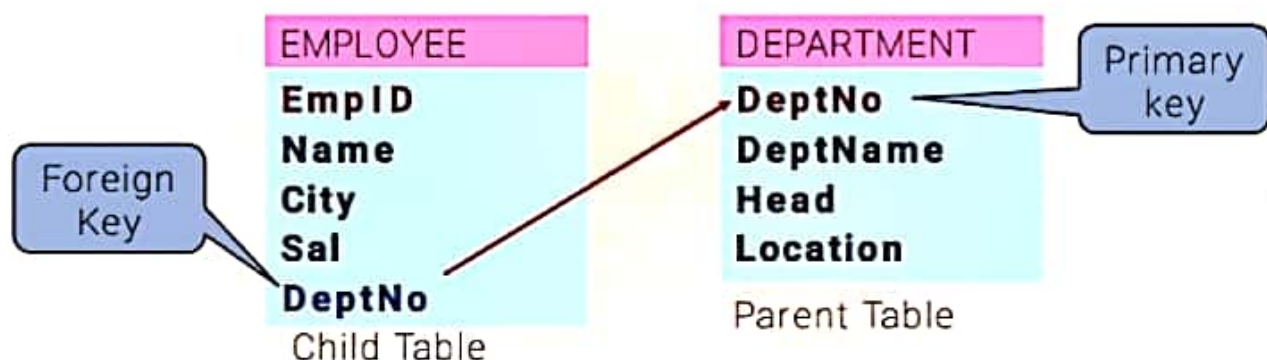
Column level constraints are defined with column definitions.

```
CREATE TABLE EMP ( Code char(3) NOT NULL,
  Name char(20) NOT NULL,
  City varchar(40),
  Pay Decimal(10,2),
  PRIMARY KEY (Code) );
```

Table level constraints are defined after all column definitions.

Implementing Foreign Key Constraints

- A Foreign key is non-key column in a table whose value is derived from the Primary key of some other table.
- Each time when record is inserted or updated in the table, the other table is referenced. This constraints is also called Referential Integrity Constraints.
- This constraints requires two tables in which Reference table (having Primary key) called **Parent table** and table having Foreign key is called **Child table**.



Implementing Foreign Key

Cont..

```
CREATE TABLE Department
( DeptNo char(2) NOT NULL PRIMARY KEY,
  DeptName char(10) NOT NULL,
  Head char(30) );
```

Parent table

```
CREATE TABLE Employee
( EmpNo char(3) NOT NULL PRIMARY KEY,
  Name char(30) NOT NULL,
  City char(20),
  Sal decimal(8,2),
  DeptNo char(2),
  FOREIGN KEY (DeptNo) REFERENCES Department (DeptNo));
```

Child Table in which Foreign key is defined.

Parent table and column to be referenced

- ❖ A Table may have multiple Foreign keys.
- ❖ Foreign key may have repeated values i.e. Non-Key Column

Modifying Table Constraints

□ Adding new column and Constraints

```
ALTER TABLE <Table Name>
```

```
ADD <Column> [<data type> <size>] [<Constraints>]
```

```
mysql> ALTER TABLE Student ADD (TelNo Integer);
```

```
mysql> ALTER TABLE Student ADD (Age Integer CHECK (Age>=5));
```

```
mysql> ALTER TABLE Emp ADD Sal Number(8,2) DEFAULT 5000 ;
```

```
mysql> ALTER TABLE Emp ADD PRIMARY KEY (EmpID);
```

```
mysql> ALTER TABLE Emp ADD PRIMARY KEY (Name,DOB);
```

□ Modifying Existing Column and Constraints

```
ALTER TABLE <Table Name>
```

```
MODIFY <Column> [<data type> <size>] [<Constraints>]
```

```
mysql> ALTER TABLE Student MODIFY Name VARCHAR(40);
```

```
mysql> ALTER TABLE Emp MODIFY (Sal DEFAULT 4000 );
```

```
mysql> ALTER TABLE Emp MODIFY (EmpName NOT NULL);
```

Modifying Table Constrains cont..

❑ Removing Column & Constraints

**ALTER TABLE <Table Name>
DROP <Column name> | <Constraints>**

```
mysql> ALTER TABLE Student DROP TelNo;
```

```
mysql> ALTER TABLE Emp DROP JOB, DROP Pay;
```

```
mysql> ALTER TABLE Student DROP PRIMARY KEY;
```

❑ Changing Column Name of Existing Column

**ALTER TABLE <Table Name>
CHANGE <Old name> <New Definition>**

```
mysql> ALTER TABLE Student  
CHANGE Name Sname Char(40);
```

Viewing & Disabling Constraints

❑ To View the Constraints

The following command will show all the details like columns definitions and constraints of EMP table.

```
mysql> SHOW CREATE TABLE EMP;
```

Alternatively you can use **DESC**cribe command:

```
mysql> DESC EMP;
```

❑ Enabling / Disabling Foreign Key Constraint

✓ You may enable or disable Foreign key constraints by setting the value of FOREIGN_KEY_CHECKS variable.

✓ You can't disable Primary key, however it can be dropped (deleted) by Alter Table... command.

▪ To Disabling Foreign Key Constraint

```
mysql> SET FOREIGN_KEY_CHECKS = 0;
```

▪ To Enable Foreign Key Constraint

```
mysql> SET FOREIGN_KEY_CHECKS = 1;
```

Grouping Records in a Query

- Some time it is required to apply a Select query in a group of records instead of whole table.
- You can group records by using **GROUP BY <column>** clause with Select command. A group column is chosen which have non-distinct (repeating) values like City, Job etc.
- Generally, the following Aggregate Functions [MIN(), MAX(), SUM(), AVG(), COUNT()] etc. are applied on groups.

Name	Purpose
SUM()	Returns the sum of given column.
MIN()	Returns the minimum value in the given column.
MAX()	Returns the maximum value in the given column.
AVG()	Returns the Average value of the given column.
COUNT()	Returns the total number of values/ records as per given column.

Aggregate Functions & NULL Values

Consider a table Emp having following records as-

Emp		
Code	Name	Sal
E1	Ram Kumar	NULL
E2	Suchitra	4500
E3	Yogendra	NULL
E4	Sushil Kr	3500
E5	Lovely	4000

Aggregate function ignores NULL values i.e. NULL values does not play any role in calculations.

```
mysql> Select Sum(Sal) from EMP;    ⇨ 12000
mysql> Select Min(Sal) from EMP;    ⇨ 3500
mysql> Select Max(Sal) from EMP;    ⇨ 4500
mysql> Select Count(Sal) from EMP;  ⇨ 3
mysql> Select Avg(Sal) from EMP;    ⇨ 4000
mysql> Select Count(*) from EMP;    ⇨ 5
```

Aggregate Functions & Group

An Aggregate function may applied on a column with **DISTINCT** or **ALL** keyword. If nothing is given **ALL** is assumed.

❑ Using **SUM (<Column>)**

This function returns the sum of values in given column or expression.

```
mysql> Select Sum(Sal) from EMP;
mysql> Select Sum(DISTINCT Sal) from EMP;
mysql> Select Sum (Sal) from EMP where City='Kanpur';
mysql> Select Sum (Sal) from EMP Group By City;
mysql> Select Job, Sum(Sal) from EMP Group By Job;
```

❑ Using **MIN (<column>)**

This functions returns the Minimum value in the given column.

```
mysql> Select Min(Sal) from EMP;
mysql> Select Min(Sal) from EMP Group By City;
mysql> Select Job, Min(Sal) from EMP Group By Job;
```

Aggregate Functions & Group

❑ Using **MAX (<Column>)**

This function returns the Maximum value in given column.

```
mysql> Select Max(Sal) from EMP;
mysql> Select Max(Sal) from EMP where City='Kanpur';
mysql> Select Max(Sal) from EMP Group By City;
```

❑ Using **AVG (<column>)**

This functions returns the Average value in the given column.

```
mysql> Select AVG(Sal) from EMP;
mysql> Select AVG(Sal) from EMP Group By City;
```

❑ Using **COUNT (< * |column>)**

This functions returns the number of rows in the given column.

```
mysql> Select Count (*) from EMP;
mysql> Select Count(Sal) from EMP Group By City;
mysql> Select Count(*), Sum(Sal) from EMP Group By Job;
```

Aggregate Functions & Conditions

You may use any condition on group, if required. HAVING <condition> clause is used to apply a condition on a group.

```
mysql> Select Job, Sum(Pay) from EMP
        Group By Job HAVING Sum(Pay)>=8000;
mysql> Select Job, Sum(Pay) from EMP
        Group By Job HAVING Avg(Pay)>=7000;
mysql> Select Job, Sum(Pay) from EMP
        Group By Job HAVING Count(*)>=5;
mysql> Select Job, Min(Pay),Max(Pay), Avg(Pay) from EMP
        Group By Job HAVING Sum(Pay)>=8000;
mysql> Select Job, Sum(Pay) from EMP Where City='Dehradun'
        Group By Job HAVING Count(*)>=5;
```

'Having' is used with Group By Clause only.



Where clause works in respect of whole table but Having works on Group only. If Where and Having both are used then Where will be executed first.

Displaying Data from Multiple Tables - Join Query

Some times it is required to access the information from two or more tables, which requires the Joining of two or more tables. Such query is called Join Query.

MySQL facilitates you to handle Join Queries. The major types of Join is as follows-

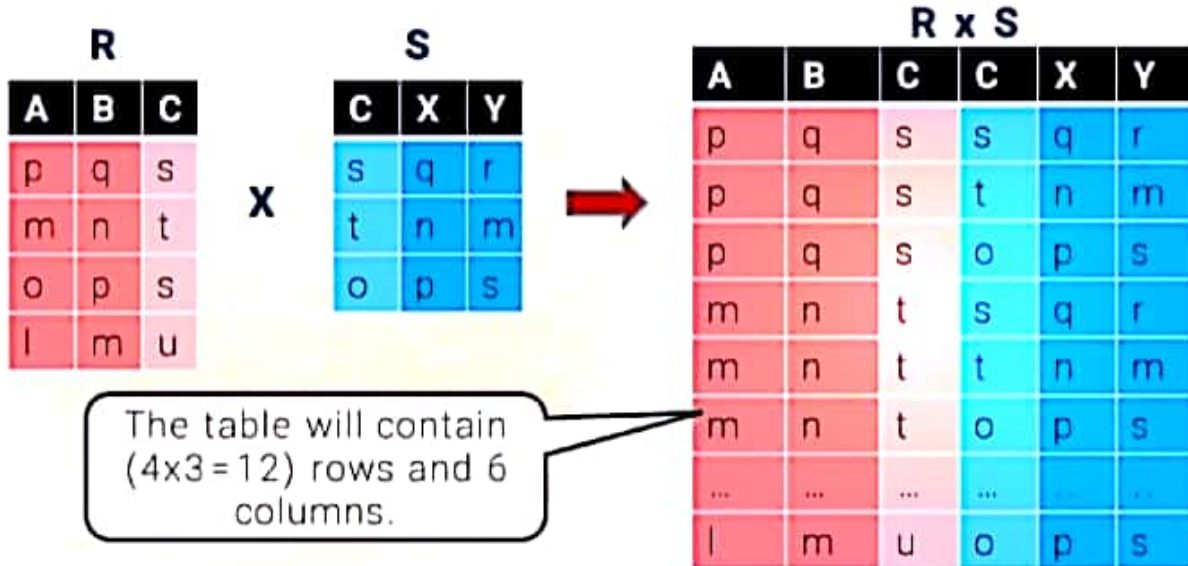
- Cross Join (Cartesian Product)**
- Equi Join**
- Non-Equi Join**
- Natural Join**

Cross Join – Mathematical Principle

Consider the two set $A = \{a,b\}$ and $B = \{1,2\}$

The Cartesian Product i.e. $A \times B = \{(a,1) (a,2) (b,1) (b,2)\}$

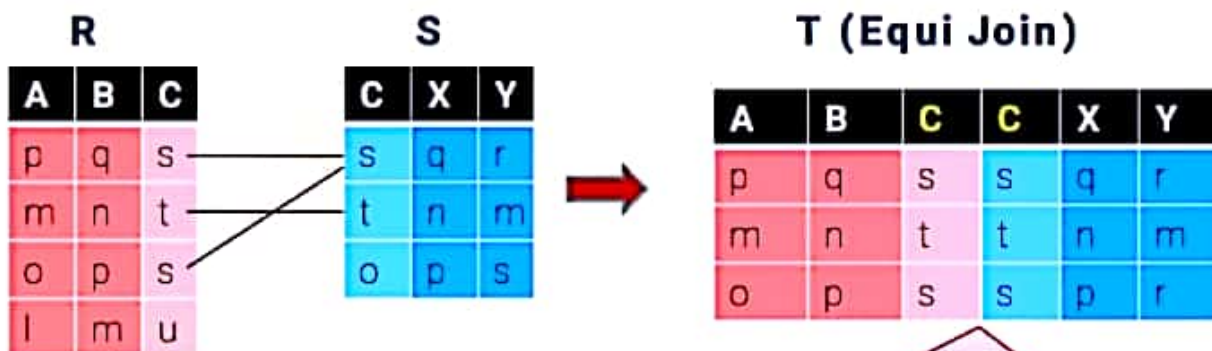
Similarly, we may compute Cross Join of two tables by joining each Record of first table with each record of second table.



Equi Join – Mathematical Principle

In Equi Join, records are joined on the equality condition of Joining Column. Generally, the Join column is a column which is common in both tables.

Consider the following table R and S having C as Join column.

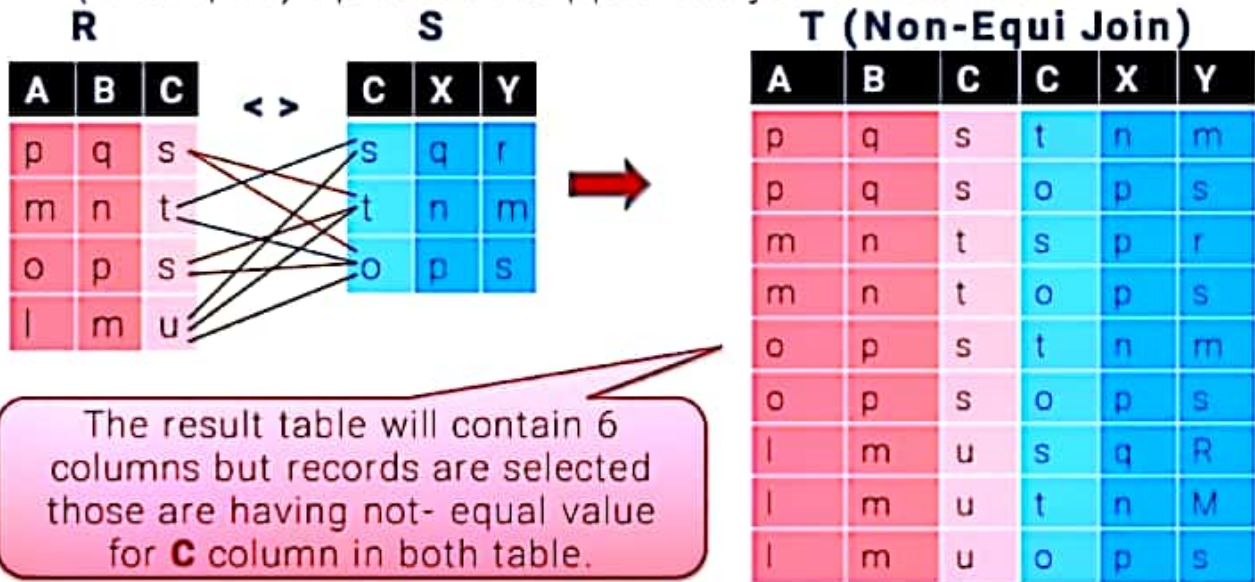


The result table will contain 6 columns but records are selected those are having Equal value for C column in both table.

Non-Equi Join – Mathematical Principle

In Non-Equi Join, records are joined on the condition other than Equal operator ($>$, $<$, $<>$, $>=$, $<=$) for Joining Column (common column).

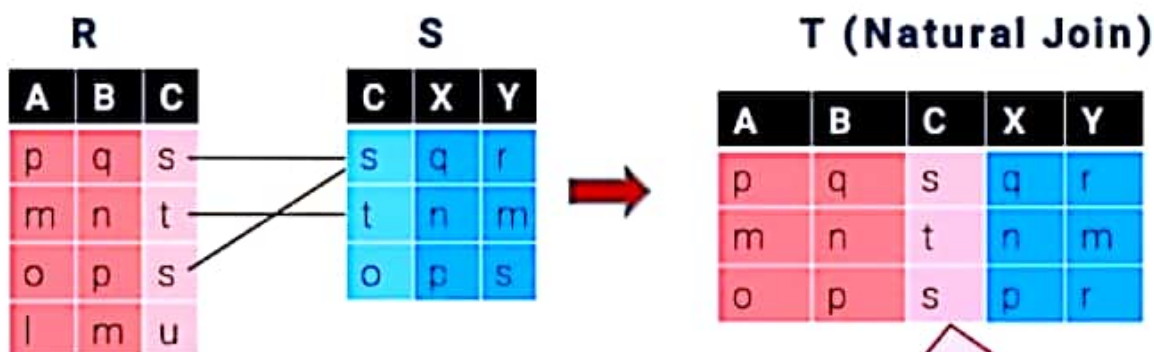
Consider the following table **R** and **S** having **C** as Join column and $<>$ (not equal) operator is applied in join condition.



Natural Join – Mathematical Principle

The Natural Join is much similar to Equi Join i.e. records are joined on the equality condition of Joining Column except that the common column appears one time.

Consider the following table **R** and **S** having **C** as Join column.



Implementing Join Operation in MySQL

Consider the two tables EMP and DEPT -

Foreign Key

Primary Key

EmpID	EName	City	Job	Pay	DeptNo
E1	Amitabh	Mumbai	Manager	50000	D1
E2	Sharukh	Delhi	Manager	40000	D2
E3	Amir	Mumbai	Engineer	30000	D1
E4	Kimmi	Kanpur	Operator	10000	D2
E4	Puneet	Chennai	Executive	18000	D3
E5	Anupam	Kolkatta	Manager	35000	D3
E6	Syna	Banglore	Secretary	15000	D1
...

EMP

DEPT

Primary Key

DeptNo	DName	Location
D1	Production	Mumbai
D2	Sales	Delhi
D3	Admn	Mumbai
D4	Research	Chennai

Suppose we want complete details of employees with their Deptt. Name and Location... this query requires the join of both tables

How to Join ?

MySQL offers different ways by which you may join two or more tables.

❑ Method 1 : Using Multiple table with FROM clause

The simplest way to implement JOIN operation, is the use of multiple table with FROM clause followed with Joining condition in WHERE clause.

```
Select * From EMP, DEPT  
Where Emp.DeptNo = Dept.DeptNo ;
```

To avoid ambiguity you should use Qualified name i.e. <Table>.<column>

If common column are differently spelled then no need to use Qualified name.

❑ Method 2: Using JOIN keyword

MySQL offers JOIN keyword, which can be used to implement all type of Join operation.

```
Select * From EMP JOIN DEPT ON Emp.DeptNo=Dept.DeptNo ;
```

Using Multiple Table with FROM clause

The General Syntax of Joining table is-

```
SELECT < List of Columns > FROM <Table1, Table 2, ... >  
WHERE <Joining Condition > [Order By ..] [Group By ..]
```

- ❑ You may add more conditions using AND/OR NOT operators, if required.
- ❑ All types of Join (Equi, No-Equi, Natural etc. are implemented by changing the Operators in Joining Condition and selection of columns with SELECT clause.

Ex. Find out the name of Employees working in Production Deptt.

```
Select Ename From EMP, DEPT  
Where Emp.DeptNo=Dept.DeptNo AND Dname='Production';
```

Ex. Find out the name of Employees working in same city from where they belongs (hometown).

```
Select Ename From EMP, DEPT  
Where Emp.DeptNo=Dept.DeptNo And City=Location;
```

Using JOIN keyword with FROM clause

MySQL 's JOIN Keyword may be used with From clause.

```
SELECT < List of Columns >  
FROM <Table1 > JOIN <Table2 > ON <Joining Condition >  
[WHERE <Condition >] [Order By ..] [Group By ..]
```

Ex. Find out the name of Employees working in Production Deptt.

```
Select Ename From EMP JOIN DEPT ON Emp.DeptNo=Dept.DeptNo  
Where Dname='Production';
```

Ex. Find out the name of Employees working in same city from where they belongs (hometown) .

```
Select Ename From EMP JOIN DEPT ON Emp.DeptNo = Dept.DeptNo  
WHERE City=Location;
```

Nested Query (A query within another query)

Sometimes it is required to join two sub-queries to solve a problem related to the single or multiple table. Nested query contains multiple query in which inner query evaluated first.

The general form to write Nested query is-

Select ... From <Table>

Where <Column1> <Operator>

(Select Column1 From <Table> [Where <Condition>])

Ex. Find out the name of Employees working in Production Deptt.

Select Ename From EMP

**Where DeptNo = (Select DeptNo From DEPT Where
DName='Production');**

Ex. Find out the name of Employees who are getting more pay than 'Ankit'.

Select Ename From EMP

Where Pay >= (Select Pay From EMP Where Ename='Ankit');

Union of Tables

Sometimes it is required to combine all records of two tables without having duplicate records. The combining records of two tables is called UNION of tables.

UNION Operation is similar to UNION of Set Theory.

E.g. If set A = {a,c,m,p,q} and Set B = {b,m,q,t,s}

Then AUB = {a,c,m,p,q,b,t,s}

[All members of Set A and Set B are taken without repeating]

Select ... From <Table1> [Where <Condition>]

UNION [ALL]

Select ... From <Table2> [Where <Condition>];

Ex. **Select Ename From PROJECT1**

UNION

Select Ename From PROJECT2 ;

Both tables or output of queries must be UNION compatible i.e. they must be same in column structure (number of columns and data types must be same).

CBSE Class -XII Accountancy
Revision Notes
Chapters-3 Part - B
Tools for financial statement analysis

The various tools used for analysis of financial statements are :

Comparative Statement : Financial Statements of two years are compared and changes in absolute terms and in percentage terms are calculated. It is a form of Horizontal Analysis.

Common Size statement : Figures of Financial statements are converted it to percentage with respect to some common base.

In Common size Income Statement **Sales/Revenue from Operations** is taken is common base where as in Common size Balance Sheet **Total assets** or **Total Equity and Liabilities** are taken as common base.

Ratio Analysis : It is a technique of Study of relationship between various items in the Financial Statements. There are mainly four types of ratios-

- 1) liquidity ratio
- 2) solvency ratio
- 3) activity ratio
- 4) profitability ratio

Cash Flow Statement : It is a statement that shows the inflow and outflow of cash and cash equivalents during a particular period which helps in finding out the causes of changes in cash position between the two balance sheet dates. It is prepared under accounting standard 3

Comparative Statements

It is a statement that shows changes in each item of the financial statement in absolute amount and in percentage, taking the amounts of the preceding as counting period as the base.

Types of Comparative Statements :

1. Comparative Balance Sheet; and
2. Comparative Statement of Profit and Loss.

Comparative Balance Sheet : It shows the increases and decreases in various items of assets, equity and liabilities in absolute term and in percentage term by taking the corresponding figures in the previous year's balance sheet as a base.

Format for a Comparative Balance Sheet

Comparative Balance Sheet of Ltd.

As at 31st March 2014 and 2015

Particulars	2014 Rs. (previous year)	2015 Rs (current year).	Absolute Change Rs. (current year- previous year)	Percentage Change %
1. EQUITY AND LIABILITIES (1) Shareholders' funds (a) Share capital (b) Reserves and surplus (2) Non-current Liabilities (a) Long-term borrowings (b) Other Long term liabilities (c) Long-term provisions (3) Current liabilities (a) Short-term borrowings (b) Trade payables (c) Other Current liabilities (d) Short-term provisions				

Total				
II. ASSETS				
(1) Non-current assets				
(a) Fixed assets				
(b) Non-current investments				
(c) Long-term loans and advances				
(2) Current Assets				
(a) Current investments				
(b) Inventories				
(c) Trade receivables				
(d) Cash and cash equivalents				
(e) Short term loans and advances				
(f) Other current assets				
Total				

*Percentage change = absolute change/ previous year *100

for example -

particulars	note no	2016 (A)	2017 (B)	absolute change C= B-A	percentage C/A*100
share holder fund		500000	300000	200000	40
current liabilities		30000	20000	10000	50
total liabilities		530000	320000	210000	40.38
assets					
fixed assets		220000	200000	20000	9.09
current assets		310000	120000	190000	61.29
total assets		530000	320000	210000	40.38

COMPARATIVE STATEMENT OF PROFIT AND LOSS/COMPARATIVE INCOME STATEMENT

Comparative Income Statement: It shows the increases and decreases in various items of income Statement in absolute amount and in percentage amount by taking the

corresponding figures in the previous year's Income Statement as a base.

Format for a Comparative Statement of Profit and Loss

Comparative Statement of Profit and Loss

For the years ended on 31st March, 2014 and 2015

Particulars	2014 Rs. (previous year)	2015 Rs. (current year)	Absolute Change Rs. (current year- previous year)	Percentage Change %
I. Revenue form operations				
II. Other Income				
III. Total Revenue (I+II)				
IV. Expenses :				
a. Cost of Material consumed				
b. Purchases of Stock-in-Trade				
c. Changes in Inventories of Finished Goods, Work-in-progress and Stock-in-trade				
d. Employees Benefit Expenses				
e. Finance Cost				
f. Depreciation & Amortisation Expenses				
g. Other Expenses				
Total Expenses				
V. Profit before Tax (III-IV)				
Less : Income Tax				
VII. Profit after Tax				

percentage = $\frac{\text{absolutechange}}{\text{previous year}} \times 100$

Importance of Comparative Statement

To make the data simple and more understandable.

To indicate the trend with respect to the previous year.

To compare the firm performance with the performance of other firm in the same business.

PARTICULARS	2016 (A)	2017 (B)	ABSOLUTE CHANGE (B-A)	PERCENTAGE C/A*100
revenue from operation	10,00,000	30,00,000	20,00,000	200
total income (A)	10,00,000	30,00,000	20,00,000	200
cost of production	2,00,000	3,00,000	1,00,000	50
other expenses	1,00,000	2,00,000	1,00,000	100
total expenses(B)	3,00,000	5,00,000	2,00,000	66.7
profit (A-B)	7,00,000	25,00,000	18,00,000	257.14
-TAX	(1,00,000)	(5,00,000)	4,00,000	400
PROFIT AFTER TAX	6,00,000	20,00,000	14,00,000	233.3

Common Size Statement

Common Size Financial Statements are the statements in which amounts of the various items of financial statements are converted into percentages to a common base.

Types of Common Size statements :

1. Common Size Balance sheet; and
2. Common Size Statement of Profit and Loss.

Common Size Balance sheet : It is a statement in which every item of assets, equity and liabilities is expressed as a percentage to the total of all assets or to the total of Equity and Liabilities.

Format for a Common Size Balance Sheet :

Common Size Balance Sheet of.....Ltd.

As at 31st March, 2014 and 2015

Particulars	Absolute Amounts	Percentage of Balance
-------------	------------------	-----------------------

	Sheet Total			
	2014 Rs.	2015 Rs.	2014 %	2015 %
1. EQUITY AND LIABILITIES				
(1) Shareholders' funds				
Share capital				
Reserves and surplus				
(3) Non-current Liabilities				
Long-term borrowings				
Other Long term liabilities				
Long-term provisions				
(4) Current liabilities				
Short-term borrowings				
Trade payables				
Other Current liabilities				
Short-term provisions				
Total				
II. ASSETS				
(1) Non-current assets				
Fixed assets				
Non-current investments				
Long-term loans and advances				
(2) Current Assets				
Current investments				
Inventories				
Trade receivables				
Cash and cash equivalents				
Short term loans and advances				
Other current assets				
Total				

note - all the items are divided by the total of balance sheet to calculate the percentage.

particulars	note no	2016 (A)	2017 (B)	PERCENTAGE 2016 (divide by total 530000)	percentage 2017 (divide by total 320000)
share holder fund		500000	300000	94.3	93.75
current liabilities		30000	20000	5.7	6.25
total liabilities		530000	320000	100	100
assets					
fixed assets		220000	200000	41.50	62.5
current assets		310000	120000	58.49	37.5
total assets		530000	320000	100	100

Common Size Income Statement or Statement of Profit and Loss: It is a statement in which every item of Statement of Profit and Loss is expressed as a percentage to the amount of Revenue from Operations.

Format for a Common Size Statement of Profit and Loss:

Common Size Statement of Profit and Loss

For the years ended on 31st March, 2014 and 2015

Particulars	Absolute Amounts		Percentage of Revenue from operation (Net Sales)	
	2014 Rs.	2015 Rs.	2014 Rs.	2015 Rs.
I. Revenue from operations				
II. Add : Other Income				

III. Total Revenue (I+II)				
IV. Expenses :				
a. Cost of Material consumed				
b. Purchases of Stock-in-Trade				
c.Changes in Inventories of Finished Goods, Work-in-progress and Stock-in-trade				
d. Employees Benefit Expenses				
e. Finance Cost				
f. Depreciation				
g. Other Expenses				
Total Expenses				
V. Profit before Tax (III-IV)				
Less : Income Tax				
VII. Profit after Tax				

note- all the items are divided by revenue from operations of that year to calculate the percentages.

PARTICULARS	2016 (A)	2017 (B)	PERCENTAGE 2016 (divide by 10,00,000)	PERCENTAGE 2017 (divide by 30,00,000)
revenue from operation	10,00,000	30,00,000	100	100
total income (A)	10,00,000	30,00,000	100	100
cost of production	2,00,000	3,00,000	20	10
other expenses	1,00,000	2,00,000	10	6.67
total expenses(B)	3,00,000	5,00,000	30	16.67
profit (A-B)	7,00,000	25,00,000	70	83.3
-TAX	(1,00,000)	(5,00,000)	10	16.67
PROFIT AFTER TAX	6,00,000	20,00,000	60	66.67

TYPOLOGY OF QUESTIONS

UNDERSTANDING

BASIC BUILDING BLOCKS (The 3 Bs) (Practical Questions Based on Illustrations)

COMPARATIVE STATEMENT OF PROFIT & LOSS

1. Prepare Comparative Statement of Profit & Loss from the following Statement of Profit and Loss.

Particulars	Note No.	2014-15 ₹	2013-14 ₹
Revenue from Operations		67,50,000	37,50,000
Employee Benefit Expenses		47,25,000	22,50,000
Other expenses		8,10,000	7,50,000
Taxes		50%	50%

Answer:

Particulars	Revenue from Operations	Total Revenue	Total Expenses	Profit before Tax	Tax	Profit After Tax
Absolute Change (₹)	30,00,000	30,00,000	25,35,000	4,05,000	2,32,500	2,32,500
Percentage Change (%)	80	80	8.45	62	62	62

COMPARATIVE BALANCE SHEET

2. Following are the summarized Balance Sheet of Disha Ltd. Prepare a Comparative Balance Sheet.

Particulars	Note No.	31-3-2015 (₹)	31-3-2014 (₹)
1. EQUITY AND LIABILITIES			
Shareholders' Funds.			
Share Capital		30,00,000	20,00,000
Reserve and Surplus		6,00,000	4,00,000
Non-Current Liabilities			
Long Term Borrowings		25,00,000	30,00,000
Current Liabilities			
Short Term Borrowings		29,00,000	14,50,000
Total		90,00,000	68,50,000
2. ASSETS			
Non-current Assets			
Fixed Assets		50,00,000	44,50,000
Current assets			
Inventories		40,00,000	24,00,000
Total		90,00,000	68,50,000

3.34

Answer:

Particulars	Share Capital	Reserve and Surplus	Long-term Borrowings	Short-term Borrowings	Fixed Assets	Inventories
Absolute Change (₹)	10,00,000	2,00,000	(5,00,000)	14,50,000	5,50,000	16,00,000
Percentage Change (%)	50	50	(16.67)	100	12.36	66.67

COMMON SIZE STATEMENT OF PROFIT & LOSS

3. Prepare a Common Size Statement from the following Statement of Profit and Loss for the year ended 31st March 2015:

Particulars	Note No.	₹
Revenue from Operations		25,25,000
Less: Employee Benefit Expenses		17,12,000
Less: Other Expenses		3,13,000
Profit Before Tax		5,00,000

Answer:

Particulars	Revenue from Operations	Total Revenue	Employees Benefit Expenses	Other Expenses	Total Expenses	Profit before Tax
Percentage (%)	100	100	67.30	12.40	80.20	19.80

COMMON SIZE BALANCE SHEET

4. Prepare Common Size Balance Sheet of JMD Ltd. from the following Information.

Particulars	Note No.	31.3.2015 ₹	31.3.2014 ₹
I EQUITY AND LIABILITIES			
Shareholders' Funds			
Share Capital		50,00,000	22,50,000
Reserve and Surplus		15,00,000	6,00,000
Non-Current Liabilities		38,75,000	15,75,000
Current Liabilities		21,25,000	6,90,000
Total		1,25,00,000	51,15,000
II ASSETS			
Non-Current Assets		1,05,00,000	45,00,000
Current Assets		20,00,000	6,15,000
Total		1,25,00,000	51,15,000

Answer:

Particulars	Share Capital	Reserve and Surplus	Non-Current Liabilities	Current Liabilities	Non-Current Assets	Current Assets
31.3.2014	43.99	11.73	30.79	13.49	87.98	12.02
31.3.2015	40	12.00	31.00	17.00	84	16

TYPOLOGY OF QUESTIONS

UNDERSTANDING, APPLICATION & HOTS

ADDITIONAL PRACTICAL QUESTIONS (For Practice)

COMPARATIVE STATEMENT OF PROFIT & LOSS

1. Prepare Comparative Statement of Profit & Loss from the following statement of Profit & Loss:

Particulars	Note No.	2014-15 ₹	2013-14 ₹
Revenue from Operations		18,00,000	12,00,000
Employee Benefit Expenses		12,60,000	7,20,000
Other Expenses		2,16,000	2,40,000
Income Tax		50%	50%

Answer:

Particulars	Revenue from Operations	Total Expenses	Profit Before Tax	Tax	Profit After Tax
Absolute Change (₹)	6,00,000	5,16,000	84,000	42,000	42,000
Percentage Change (%)	50	34.96	35	35	35

2. From the following Statement of Profit and Loss of Moontrack Ltd., for the years ended 31st March 2011 and 2012, prepare a 'Comparative Statement of Profit & Loss.

Particulars	Note No.	2011-12 ₹	2010-11 ₹
Revenue from operations		40,00,000	24,00,000
Other Income		24,00,000	18,00,000
Expenses		16,00,000	14,00,000

[CBSE 2013 (Outside)]

Answer.

Particulars	Revenue from Operations	Other Income	Total Income	Expenses	Profit Before Tax
Absolute Change (₹)	16,00,000	6,00,000	22,00,000	2,00,000	20,00,000
Percentage Change (%)	66.7	33.3	52.4	14.3	71.4

3. Prepare Comparative Statement of Profit & Loss from the following Statement of Profit and Loss of Kuhu Ltd.

Particulars	Note No.	2014-15	2013-14
		₹	₹
Revenue from Operations		1,37,330	99,450
Employee Benefit Expenses		49,770	35,550
Other Expenses		47,730	39,450
Tax 50%		-	-

Answer:

Particulars	Revenue from Operations	Total Expenses	Profit Before Tax	Tax	Profit After Tax
Absolute Change (₹)	38,280	22,500	15,780	7,890	7,890
Percentage Change (%)	38.49	30	64.54	64.54	64.54

4. From the following Statement of Profit & Loss, prepare a Comparative Statement of Profit & Loss.

Particulars	Note No.	2014-15	2013-14
		₹	₹
Revenue from Operations		13,50,000	8,00,000
Employee Benefit Expenses		5,25,000	3,50,000
Other Expenses		1,90,000	1,75,000
Other Income		6000	9000
Taxes		75,000	50,000

Answer:

Particulars	Revenue from Operations	Other Income	Total Revenue	Total Expenses	Profit before Tax	Taxes	Profit after Tax
Absolute Change (₹)	5,50,000	(3,000)	5,47,000	1,90,000	3,57,000	25,000	3,32,000
Percentage Change (%)	68.75	(33.33)	67.61	36.19	125.70	50	141.88

5. From the following Statement of Profit & Loss details, prepare a Comparative Statement Profit & Loss.

Particulars	Note No.	2014-15 (₹)	2013-14 (₹)
		Revenue from Operations	
Employee Benefit Expenses		9,75,000	6,00,000
Other Expenses		3,45,000	3,00,000
Other Incomes		12,000	25,500
Income Tax		2,10,000	1,35,000

Answer:

Particulars	Revenue from Operations	Other Income	Total Revenue	Total Expenses	Profit before Tax	Tax	Profit after Tax
Absolute Change (₹)	9,75,000	(13,000)	9,61,500	4,20,000	5,41,500	75,000	4,66,500
Percentage Change (%)	43.33	(52.94)	42.26	46.67	39.37	55.55	37.61

6. From the following Statement of Profit & Loss, prepare Comparative Statement of Profit & Loss. You are also required to interpret the results and give suitable comments.

Particulars	Note No.	Year II ₹	Year I ₹
Revenue from Operations		30,00,000	25,00,000
Employee Benefit Expenses from Operations		35% of Revenue from operation	45% of Revenue
Other Expenses		6,80,000	5,90,000
Income Tax rate		50%	50%

Answer:

Particulars	Revenue from Operations	Total Expenses	Profit Before Tax	Taxes	Profit After Tax
Absolute Change (₹)	5,00,000	15,000	4,85,000	2,42,500	2,42,500
Percentage Change (%)	20	0.87	61.78	61.78	61.78

Interpretation and Comments

- (i) The Comparative Statement of Profit & Loss of the company reveals that there has been an increase in Revenue from Operations by ₹ 5,00,000 i.e., 20% whereas Employee Benefit Expenses has been decreased by i.e., 6.67%.
- (ii) Other expenses has been increased by ₹ 90,000 i.e., 15.25% which led to increase in Profit Before Tax by ₹ 4,85,000 i.e., 61.78%.
- The overall financial position is satisfactory.

7. From the following Statement of Profit & Loss, prepare Comparative Statement of Profit & Loss. You are also required to interpret the results and give suitable comments.

Particulars	Note No.	2014-15 ₹	2013-14 ₹
Revenue from Operations		60,00,000	54,00,000
Employee Benefit Expenses		30% of Revenue from operations	40% of Revenue from operation
Other Expenses		10,80,000	10,20,000
Income Tax rate		50%	50%

3.38

Answer.

Particulars	Revenue from Operations	Total Expenses	Profit Before Tax	Taxes	Profit After Tax
Absolute Change (₹)	6,00,000	(3,00,000)	9,00,000	4,50,000	4,50,000
Percentage Change (%)	11.11	(9.43)	40.54	40.54	40.54

Interpretation and Comments

- (i) The Comparative Statement of Profit & Loss of the company reveals that there has been an increase in Revenue from operation by ₹ 6,00,000 i.e., 11.11% whereas Employee Benefit Expenses has decreased by i.e., 16.67%.
- (ii) Other expenses has increased only by ₹ 60,000 i.e., 5.88% which led to increase in Before Tax Profit by ₹ 9,00,000 i.e., 40.54%.
- The overall financial position is satisfactory.

8. Prepare a Comparative Statement of Profit & Loss from the following :

	Note	2014-15 ₹	2013-14 ₹
Revenue from Operations		3,00,000	2,50,000
Employees Benefit Expenses		1,60,000	1,25,000
Other Expenses		19,000	16,000

Interest on investments ₹ 18,000 and taxes payable @ 50%.

Answer.

Particulars	Revenue from Operations	Other Income	Total Revenue	Total Expenses	Profit before Tax	Tax	Profit after Tax
Absolute (₹)	50,000	N/L	50,000	38,000	12,000	6,000	6,000
Percentage (%)	20	N/L	18.66	26.95	9.45	9.45	9.45

9. Prepare a Comparative Statement of Profit & Loss from the following Statement of Profit & Loss :

	Note No.	2014-15 ₹	2013-14 ₹
Revenue from Operations		6,25,000	5,00,000
Employees Benefit Expenses		3,25,000	2,50,000
Other Expenses		30,000	25,000

Interest on investments ₹ 20,000 and taxes payable @ 50%.

Answer.

Particulars	Revenue from Operations	Other Income	Total Revenue	Total Expenses	Profit before Tax	Tax	Profit after Tax
Absolute (₹)	1,25,000	NIL	1,25,000	80,000	45,000	22,500	22,500
Percentage (%)	25	NIL	24.04	29.09	18.37	18.37	18.37

10. From the following Statement of Profit & Loss prepare a Comparative Statement of Profit & Loss for the period 2013-14 and 2014-15.

	Note No	2014-15 ₹	2013-14 ₹
Revenue from Operation		12,60,000	9,00,000
Employees Benefit Expenses		6,30,000	5,40,000
Other Expenses		15% of Employee Benefit Expenses	20% of Employee Benefit Expenses
Income tax		50%	50%

Answer.

Particulars	Revenue from Operations	Total Expenses	Profit Before Tax	Taxes	Profit After Tax
Absolute Change (₹)	3,60,000	76,500	2,83,500	1,41,750	1,41,750
Percentage Change (%)	40	11.81	112.5	112.5	112.5

11. From the following Statement of Profit & Loss prepare a Comparative Statement of Profit & Loss for the period 2013-14 and 2014-15.

	Note No.	2014-15 ₹	2013-14 ₹
Revenue from Operation		7,50,000	4,00,000
Employees Benefit Expenses		3,75,000	2,40,000
Other Expenses		15% of Employee Benefit Expenses	20% of Employee Benefit Expenses
Income tax		50%	50%

Answer.

Particulars	Revenue from Operations	Total Expenses	Profit Before Tax	Taxes	Profit After Tax
Absolute Change (₹)	3,50,000	1,43,250	2,06,750	1,03,375	1,03,375
Percentage Change (%)	87.5	49.74	184.60	184.60	184.60

COMPARATIVE BALANCE SHEET

12. Following are the summarized Balance Sheets Aman Ltd. as at 31.3.2014 and 31.3.2015. Prepare a Comparative Balance Sheet.

Particulars	Note No.	31.3.2015 ₹	31.3.2014 ₹
I EQUITY AND LIABILITIES			
Shareholders' Funds		4,80,000	3,75,000
Share Capital		96,000	82,500
Reserve and Surplus			
Non-current Liabilities		4,00,000	4,87,500
Long term Borrowings			
Current Liabilities		3,84,000	2,85,000
Short term Borrowings			
Total		13,60,000	12,30,000
II ASSETS			
Non-Current Assets		7,92,000	8,55,000
Fixed Assets			
Tangible Assets			
Current Assets		5,68,000	3,75,000
Inventories			
Total		13,60,000	12,30,000

Answer.

Particulars	Share Capital	Reserve and Surplus	Long-term Borrowings	Short-term Borrowings	Tangible Assets	Current Assets
Absolute Change (₹)	1,05,000	13,500	(87,500)	99,000	(63,000)	1,99,000
Percentage Change (%)	28	16.36	(17.95)	34.74	(7.37)	51.47

13. Following are the summarized Balance Sheets Raj Ltd. as at 31.3.2014 and 31.3.2015. Prepare a Comparative Balance Sheet.

Particulars	Note No.	31.3.2015 ₹	31.3.2014 ₹
I EQUITY AND LIABILITIES			
Shareholders' Fund			
Share Capital			
Reserve and Surplus		12,00,000	10,00,000
Non-current Liabilities		2,40,000	2,20,000
Long term Borrowings			
Current Liabilities		10,00,000	13,00,000
Short term Borrowings			
Total		9,60,000	7,60,000
		34,00,000	32,80,000
II ASSETS			
Non-Current Assets			
Fixed Assets			
Current Assets		19,80,000	22,80,000
Inventories			
Total		14,20,000	10,00,000
		34,00,000	32,80,000

Answer:

Particulars	Share Capital	Reserve and Surplus	Long-term Borrowings	Short-term Borrowings	Fixed Assets	Inventories
Absolute Change (₹)	2,00,000	20,000	(3,00,000)	2,00,000	(3,00,000)	4,20,000
Percentage Change (%)	20	9.09	(23.08)	26.32	(13.16)	42.00

14. Prepare the Comparative Balance of M/s Shyam & Co. Ltd. from the following Balance Sheets as at 31.3.2014 and 31.3.2015:

Particulars	Note No.	31.3.2015 ₹	31.3.2014 ₹
EQUITY AND LIABILITIES			
Shareholders' Funds			
Share Capital		3,20,000	2,50,000
Reserves and Surplus		28,000	20,000
Non-Current Liabilities			
Long Term Borrowings		1,60,000	2,00,000
Total		<u>5,08,000</u>	<u>4,70,000</u>
ASSETS			
Non-Current-Assets			
Fixed Assets		3,44,000	4,00,000
Current Assets			
Trade Receivables		28,000	22,500
Cash and Cash Equivalents		1,36,000	47,500
Total		<u>5,08,000</u>	<u>4,70,000</u>

Answer:

Particulars	Share Capital	Reserve and Surplus	Long-term Borrowings	Fixed Assets	Trade Receivables	Cash and Cash Equivalents
Absolute Change (₹)	70,000	8,000	(40,000)	(56,000)	5,500	88,500
Percentage Change (%)	28	40	(20)	(14)	24.44	186.32

15. Prepare the Comparative Balance Sheet of M/s Mickey Company Ltd. from the following Balance Sheet as at 31.3.2014 and 31.3.2015.

3.42

Particulars	Note No.	31.3.2015 ₹	31.3.2014 ₹
EQUITY AND LIABILITIES			
Shareholders' Funds			
Share Capital		11,20,000	8,00,000
Reserves and Surplus		98,000	64,000
Non-Current Liabilities			
Long Term Borrowings		5,60,000	6,40,000
Total		<u>17,78,000</u>	<u>15,04,000</u>
ASSETS			
Non-Current-Assets			
Fixed Assets		12,04,000	12,80,000
Current Assets			
Trade Receivables		98,000	72,000
Cash and Cash Equivalents		4,76,000	1,52,000
Total		<u>17,78,000</u>	<u>15,04,000</u>

Answer:

Particulars	Share Capital	Reserve and Surplus	Long-term Borrowings	Fixed Assets	Trade Receivables	Cash and Cash Equivalents
Absolute Change (₹)	3,20,000	34,000	(80,000)	(76,000)	25,000	2,45,000
Percentage Change (%)	40	53.125	(12.5)	(5.9)	34.72	161.18

COMMON SIZE INCOME STATEMENT

16. Prepare a Common Size Income Statement from the following Statement of Profit & Loss

STATEMENT OF PROFIT & LOSS

for the year ended 31st March 2015

Particulars	Note No.	₹
Revenue from Operations		30,60,000
Less: Employee Benefit Expenses		22,80,000
Less: Other Expenses		2,08,000
Profit		<u>5,72,000</u>

Answer:

Particulars	Revenue from Operations	Employees Benefit Expenses	Other Expenses	Profit
Percentage of Revenue from Operations	100	74.51	6.80	18.69

17. Prepare a Common Size Statement of Profit & Loss from the following Statement of Profit & Loss.

STATEMENT OF PROFIT & LOSS
for the year ended 31st March 2015

Particulars	Note No.	₹
Revenue from Operations		20,30,000
Less: Employee Benefit Expenses		16,88,000
Less: Other Expenses		1,63,700
Profit		1,78,300

Answer:

Particulars	Revenue from Operations	Employees Benefit Expenses	Other Expenses	Profit
Percentage of Revenue from Operations	100	83.16	8.06	8.78

18. From the following Statement of Profit & Loss of ABC prepare a Common Size Statement of Profit & Loss.

Particulars	Note No.	2014-15	2013-14
Revenue from Operations		22,50,000	17,50,000
Other Income		3,25,000	2,50,000
Employees Benefit Expenses		8,25,000	4,50,000
Income tax		30%	30%

Answer.

Particulars	Revenue from Operations	Other Income	Total Revenue	Employees Benefit Expenses	Profit before Tax	Tax	Profit after Tax
2013-14	100	14.29	114.29	25.71	88.57	26.57	62
2014-15	100	14.44	114.44	36.67	77.78	23.33	54.44

COMMON SIZE BALANCE SHEET

19. Prepare Common Size Balance Sheet of PQR Ltd. from the following information.

Particulars	Note No.	31.3.2015 ₹	31.3.2014 ₹
EQUITY AND LIABILITIES			
Shareholders' Funds			
Share Capital		6,00,000	5,00,000
Reserves and Surplus		1,00,000	1,00,000
Non-Current Liabilities			
Long term Borrowings		1,50,000	1,20,000
Current Liabilities			
Short term Liabilities		3,40,000	2,70,000
Total		<u>11,90,000</u>	<u>9,90,000</u>

3.44

ASSETS		
Non-Current Assets		
Fixed Assets	4,50,000	4,00,000
Tangible Assets	4,00,000	3,00,000
Intangible Assets	2,00,000	2,00,000
Non-Current Investments		
Current Assets	1,40,000	90,000
Inventories	11,90,000	9,90,000
Total		

Answer.

Particulars	Share Capital	Reserve and Surplus	Long-term Borrowings	Short-term Borrowings	Tangible Assets	Intangible Assets	Non-Current Investment	Inventory
31.3.2014	50.51	10.10	12.12	27.27	40.40	30.30	20.20	9.10
31.3.2015	50.42	8.40	12.61	28.57	37.82	33.61	16.81	11.76

Hint

All percentages have been calculated on the basis of total of Balance Sheet. For the year ending 31st March 2014 percentages have been based on ₹ 9,90,000 and in for the year ending 31st March 2015 percentages have been based on ₹ 11,90,000.

20. Prepare Common Size Balance Sheet of Daljeet Ltd. from the following information.

Particulars	Note No.	31.3.2015 ₹	31.3.2014 ₹
EQUITY AND LIABILITIES			
Shareholders' Funds			
Share Capital			
Reserves and Surplus		24,50,000	18,00,000
Non-Current Liabilities		10,00,000	10,00,000
Long term Borrowings			
Current Liabilities		18,00,000	15,00,000
Short term Liabilities			
Total		19,50,000	12,50,000
ASSETS		72,00,000	55,50,000
Non-Current Assets			
Fixed Assets			
Tangible Assets			
Intangible Assets		25,50,000	17,00,000
Non-Current Investments		25,00,000	17,00,000
Current Assets		19,00,000	17,00,000
Inventories			
Total		2,50,000	4,50,000
		72,00,000	55,50,000

Answer.

Particulars	Share Capital	Reserve and Surplus	Long-term Borrowings	Short-term Borrowings	Tangible Assets	Intangible Assets	Non-Current Investment	Inventory
31.3.2014	32.43	18.02	27.03	22.52	30.63	30.63	30.63	8.1
31.3.2015	34.03	13.89	25	27.08	35.42	34.72	26.39	3.47

Hint

All percentages have been calculated on the basis of total of Balance Sheet. For the year ending 31st March 2014 percentages have been based on ₹ 55,50,000 and in for the year ending 31st March 2015 percentages have been based on ₹ 72,00,000.

FILL IN THE MISSING INFORMATION/FIGURES

21. (Comparative Statement of Profit & Loss) Fill in the missing information in the following Comparative Statement of Profit and Loss.

COMPARATIVE STATEMENT OF PROFIT AND LOSS
for the year ended 31st March 2014 and 2015.

S. No.	Particulars	Note No.	2013-14	2014-15	Absolute changes Increase or Decrease (B) – (A)	Percent changes Increase or Decrease $\frac{C}{A} \times 100$
			₹	₹	₹	₹
			(A)	(B)	(C)	(D)
1.	Revenue from Operations		25,50,000	28,00,000	2,50,000	9.80
2.	Expenses:					
	(a) Employee Benefit Expenses	
	(b) Other Expenses		50,000	80,000
	Total Expenses	
3.	Profit Before Taxes (1 – 2)	
4.	Taxes (40%)		6,00,000	7,56,000
5.	Profit After Taxes (3 – 4)	

WEEK-4 (H.H.W)
ST. MARY'S PUBLIC SCHOOL



Mathematics

CLASS - XII

SOLUTIONS

OF

N.C.E.R.T

(EX.4.1 TO EX.4.6)

INCLUDING SELF EVALUATION TEST(10 MARKS)

CHAPTER

DETERMINANTS

(DO IN THE REGISTER: 30+10 MARKS)

Mathematics

(Chapter - 4) (Determinants)

(Class 12)

Exercise 4.1

Evaluate the determinants in Exercises 1 and 2.

Question 1:

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$$

Answer 1:

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} \quad \text{Expanding along } R_1, \text{ we get} \\ = 2 \times (-1) - 4 \times (-5) = -2 + 20 = 18$$

Question 2:

$$(i) \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$$

$$(ii) \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$$

Answer 2:

$$(i) \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = \cos \theta \times \cos \theta - \sin \theta \times (-\sin \theta) = \cos^2 \theta + \sin^2 \theta = 1$$

$$(ii) \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = (x^2 - x + 1) \times (x + 1) - (x - 1) \times (x + 1) \\ = x^3 + x^2 - x^2 - x + x + 1 - (x^2 + x - x - 1) \\ = x^3 - x^2 + 2$$

Question 3:

If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Answer 3:

$$|2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} \quad \text{Expanding along } R_1, \text{ we get} \\ = 2 \times 4 - 4 \times 8 = 8 - 32 = -24 \quad \dots (1)$$

$$4|A| = 4 \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} \quad \text{Expanding along } R_1, \text{ we get} \\ = 4(1 \times 2 - 2 \times 4) = 4(-6) = -24 \quad \dots (2)$$

From the equation (1) and (2), we get, $|2A| = 4|A|$

Question 4:

If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$

Answer 4:

$$|3A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix} \quad \text{Expanding along } R_1, \text{ we get} \\ = 3(36 - 0) - 0(0 - 0) + 1(0 - 0) = 108 \quad \dots (1)$$

$$27|A| = 27 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix} \quad \text{Expanding along } R_1, \text{ we get} \\ = 27\{1(4 - 0) - 0(0 - 0) + 1(0 - 0)\} = 27(4) = 108 \quad \dots (2)$$

From the equation (1) and (2), we get, $|3A| = 27|A|$

(Class 12)

Question 5:

Evaluate the determinants:

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ (ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ (iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ (iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Answer 5:

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ Expanding along R_1 , we get
 $= 3(0 - 5) + 1(0 + 3) - 2(0 - 0) = -15 + 3 - 0 = -12$

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ Expanding along R_1 , we get
 $= 3(1 + 6) + 4(1 + 4) + 5(3 - 2) = 21 + 20 + 5 = 46$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ Expanding along R_1 , we get
 $= 0(0 + 9) - 1(0 - 6) + 2(-3 - 0) = 0 + 6 - 6 = 0$

(iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ Expanding along R_1 , we get
 $= 2(0 - 5) + 1(0 + 3) - 2(0 - 6) = -10 + 3 + 12 = 5$

Question 6:

If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find $|A|$.

Answer 6:

$|A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$ Expanding along R_1 , we get
 $= 1(-9 + 12) - 1(-18 + 15) - 2(8 - 5) = 3 + 3 - 6 = 0$

Question 7:

Find values of x , if

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Answer 7:

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$
 $\Rightarrow 2 - 20 = 2x^2 - 24 \quad \Rightarrow x^2 = 3 \quad \Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$
 $\Rightarrow 10 - 12 = 5x - 6x \quad \Rightarrow -2 = -x \quad \Rightarrow x = 2$

(Class 12)

Question 8:

If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to:

- (A) 6 (B) ± 6 (C) -6 (D) 0

Answer 8:

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$$\Rightarrow x^2 - 36 = 36 - 36$$

$$\Rightarrow x^2 = 36$$

$$\Rightarrow x = \pm 6$$

Hence, the option (B) is correct.

Exercise 4.2

Using the property of determinants and without expanding in Exercises 1 to 7, prove that:

Question 1:

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$$

Answer 1:

$$\text{LHS} = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = \begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix} \quad [\text{Applying } C_1 \rightarrow C_1 + C_2]$$

$$= 0 = \text{RHS} \quad [\because C_1 = C_3]$$

Question 2:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

Answer 2:

$$\text{LHS} = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix} \quad [\text{Applying } C_1 \rightarrow C_1 + C_2 + C_3]$$

$$= 0 = \text{RHS} \quad [\because \text{In column } C_1 \text{ every element is zero.}]$$

Question 3:

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$$

Answer 3:

$$\text{LHS} = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix} \quad [\text{Applying } C_3 \rightarrow C_3 - C_1]$$

$$= 9 \begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix}$$

[Taking common 9 from C_3]

$$= 0 = \text{RHS}$$

[$\because C_2 = C_3$]

Question 4:

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

Answer 4:

$$\text{LHS} = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = \begin{vmatrix} 1 & bc & ab+bc+ca \\ 1 & ca & ab+bc+ca \\ 1 & ab & ab+bc+ca \end{vmatrix} \quad [\text{Applying } C_3 \rightarrow C_3 + C_2]$$

$$= (ab + bc + ca) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix} \quad [\text{Taking } ab + bc + ca \text{ as common from } C_1]$$

$$= 0 = \text{RHS} \quad [\because C_1 = C_3]$$

Question 5:

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Answer 5:

$$\text{LHS} = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= \begin{vmatrix} 2c & 2r & 2z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} \quad [\text{Applying } R_1 \rightarrow R_1 + R_2 - R_3]$$

$$= 2 \begin{vmatrix} c & r & z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} \quad [\text{Taking 2 as common from } R_1]$$

$$= 2 \begin{vmatrix} c & r & z \\ a & p & x \\ a+b & p+q & x+y \end{vmatrix} \quad [\text{Applying } R_2 \rightarrow R_2 - R_1]$$

$$= 2 \begin{vmatrix} c & r & z \\ a & p & x \\ b & q & y \end{vmatrix} \quad [\text{Applying } R_3 \rightarrow R_3 - R_2]$$

$$= -2 \begin{vmatrix} a & p & x \\ c & r & z \\ b & q & y \end{vmatrix} \quad [\text{Applying } R_1 \leftrightarrow R_2]$$

$$= 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \text{RHS} \quad [\text{Applying } R_2 \leftrightarrow R_3]$$

Q.NO.6 TO Q.NO. 9 TRY YOURSELF

[FOR SOLUTION WATCH MY VIDEO LESSON]

Question 10:

$$(i) \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

$$(ii) \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$$

Answer 10:

$$\begin{aligned} (i) \text{ LHS} &= \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} \\ &= \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix} \quad [\text{Applying } C_1 \rightarrow C_1 + C_2 + C_3] \\ &= (5x+4) \begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix} \quad [\text{Taking } 5x+4 \text{ as common from } C_1] \\ &= (5x+4) \begin{vmatrix} 0 & x-4 & 0 \\ 0 & 4-x & x-4 \\ 1 & 2x & x+4 \end{vmatrix} \quad [\text{Applying } R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3] \\ &= (5x+4) \{(x-4)(x-4) - (4-x)0\} \quad [\text{Expanding along } C_1] \\ &= (5x+4)(4-x)^2 = \text{RHS} \end{aligned}$$

$$\begin{aligned} (ii) \text{ LHS} &= \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} \\ &= \begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix} \quad [\text{Applying } C_1 \rightarrow C_1 + C_2 + C_3] \\ &= (3y+k) \begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix} \quad [\text{Taking } 3y+k \text{ as common from } C_1] \\ &= (3y+k) \begin{vmatrix} 0 & -k & 0 \\ 0 & k & -k \\ 1 & y & y+k \end{vmatrix} \quad [\text{Applying } R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3] \\ &= (3y+k) \{(-k)(-k) - (k)0\} \quad [\text{Expanding along } C_1] \\ &= (3y+k)k^2 = \text{RHS} \end{aligned}$$

Question 11:

$$(i) \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$(ii) \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$

Answer 11:

$$\begin{aligned} \text{(i) LHS} &= \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} \\ &= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} \quad [\text{Applying } R_1 \rightarrow R_1 + R_2 + R_3] \\ &= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} \quad [\text{Taking } a+b+c \text{ as common from } R_1] \\ &= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ a+b+c & -a-b-c & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix} \quad [\text{By } C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3] \\ &= (a+b+c)[(a+b+c)^2 - 0] \quad [\text{Expanding along } R_1] \\ &= (a+b+c)^3 = \text{RHS} \end{aligned}$$

$$\begin{aligned} \text{(ii) LHS} &= \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} \\ &= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix} \quad [\text{Applying } C_1 \rightarrow C_1 + C_2 + C_3] \\ &= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix} \quad [\text{Taking } 2(x+y+z) \text{ common from } C_1] \\ &= 2(x+y+z) \begin{vmatrix} 0 & -(x+y+z) & 0 \\ 0 & x+y+z & -(x+y+z) \\ 1 & x & z+x+2y \end{vmatrix} \quad [\text{By } R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3] \\ &= 2(x+y+z)[(x+y+z)^2 - 0] \quad [\text{Expanding along } C_1] \\ &= 2(x+y+z)^3 = \text{RHS} \end{aligned}$$

Question 12:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

Answer 12:

$$\begin{aligned} \text{LHS} &= \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix} \quad [\text{Applying } C_1 \rightarrow C_1 + C_2 + C_3] \\ &= (1+x+x^2) \begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix} \quad [\text{Taking } 1+x+x^2 \text{ as common from } C_1] \\ &= (1+x+x^2) \begin{vmatrix} 0 & x-1 & x^2-x \\ 0 & 1-x^2 & x-1 \\ 1 & x^2 & 1 \end{vmatrix} \quad [\text{Applying } R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3] \\ &= (1+x+x^2)(1-x)^2 \begin{vmatrix} 0 & -1 & -x \\ 0 & 1+x & -1 \\ 1 & x^2 & 1 \end{vmatrix} \quad [\text{Taking } 1-x \text{ as common from } R_1 \text{ and } R_2] \end{aligned}$$

$$\begin{aligned}
 &= (1+x+x^2)(1-x)^2\{1+x(1+x)\} \quad [\text{Expanding along } C_1] \\
 &= (1+x+x^2)(1-x)^2(1+x+x^2) = (1-x^3)^2 = \text{RHS}
 \end{aligned}$$

Question 13:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Answer 13:

$$\begin{aligned}
 \text{LHS} &= \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \\
 &= \frac{1}{a} \begin{vmatrix} 1+a^2-b^2 & 2ab & -2ab \\ 2ab & 1-a^2+b^2 & 2a^2 \\ 2b & -2a & a-a^3-ab^2 \end{vmatrix} \quad [\text{Applying } C_3 \rightarrow aC_3] \\
 &= \frac{1}{a} \begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2ab & 1-a^2+b^2 & 1+a^2+b^2 \\ 2b & -2a & -a-a^3-ab^2 \end{vmatrix} \quad [\text{Applying } C_3 \rightarrow C_3 + C_2] \\
 &= \frac{1+a^2+b^2}{a} \begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2ab & 1-a^2+b^2 & 1 \\ 2b & -2a & -a \end{vmatrix} \quad [\text{Taking } 1+a^2+b^2 \text{ as common from } C_3] \\
 &= \frac{1+a^2+b^2}{a^2} \begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2a^2b & a-a^3+ab^2 & a \\ 2b & -2a & -a \end{vmatrix} \quad [\text{Applying } R_2 \rightarrow aR_2]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1+a^2+b^2}{a^2} \begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2a^2b+2b & -a-a^3+ab^2 & 0 \\ 2b & -2a & -a \end{vmatrix} \quad [\text{Applying } R_2 \rightarrow R_2 + R_3] \\
 &= (1+a^2+b^2) \begin{vmatrix} 1+a^2-b^2 & 2b & 0 \\ 2a^2b+2b & -1-a^3+b^2 & 0 \\ 2b & -2 & -1 \end{vmatrix} \quad [\text{Taking } a \text{ as common from } C_2 \text{ and } C_3] \\
 &= (1+a^2+b^2)(-1)\{(1+a^2-b^2)(-1-a^3+b^2) - 2b(2a^2b+2b)\} \\
 &\quad \quad \quad [\text{Expanding along } C_3] \\
 &= -(1+a^2+b^2)(-1-a^3+b^2-a^2-a^4+a^2b^2+b^2+a^2b^2-b^4-4a^2b^2-4b^2) \\
 &= (1+a^2+b^2)(1+a^4+4+2a^2+2a^2b^2+2b^2) \\
 &= (1+a^2+b^2)(1+a^2+b^2)^2 = (1-x^3)^2 = \text{RHS}
 \end{aligned}$$

Question 14:

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

Answer 14:

$$\text{LHS} = \begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} a^3 + a & a^2b & a^2c \\ ab^2 & b^3 + b & b^2c \\ c^2a & c^2b & c^3 + c \end{vmatrix} \quad [\text{Applying } R_1 \rightarrow aR_1, R_2 \rightarrow bR_2, R_3 \rightarrow cR_3]$$

$$= \frac{abc}{abc} \begin{vmatrix} a^2 + 1 & a^2 & a^2 \\ b^2 + 1 & b^2 & b^2 \\ c^2 & c^2 & c^2 + 1 \end{vmatrix}$$

[Taking a as common from C_1 , b from C_2 and c from C_3]

$$= \begin{vmatrix} 1 + a^2 + b^2 + c^2 & 1 + a^2 + b^2 + c^2 & 1 + a^2 + b^2 + c^2 \\ \frac{b^2}{c^2} & \frac{b^2 + 1}{c^2} & \frac{b^2}{c^2 + 1} \\ c^2 & c^2 & c^2 + 1 \end{vmatrix}$$

[By $R_1 \rightarrow R_1 + R_2 + R_3$]

$$= (1 + a^2 + b^2 + c^2) \begin{vmatrix} 1 & 1 & 1 \\ \frac{b^2}{c^2} & \frac{b^2 + 1}{c^2} & \frac{b^2}{c^2 + 1} \end{vmatrix}$$

[Taking $1 + a^2 + b^2 + c^2$ as common from R_1]

$$= (1 + a^2 + b^2 + c^2) \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & \frac{b^2}{c^2 + 1} \\ 0 & -1 & c^2 + 1 \end{vmatrix} \quad [\text{Applying } C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3]$$

$$= (1 + a^2 + b^2 + c^2) \{1 - 0\} \quad [\text{Expanding along } R_1]$$

$$= 1 + a^2 + b^2 + c^2 = \text{RHS}$$

(Class 12)

Choose the correct answer in Exercises 15 and 16.

Question 15:

Let A be a square matrix of order 3×3 , then $|kA|$ is equal to:

- (A) $k|A|$ (B) $k^2|A|$ (C) $k^3|A|$ (D) $3k|A|$

Answer 15:

If B be a square matrix of order $n \times n$, then $|kB| = k^{n-1}|B|$

Therefore, $|kA| = k^{3-1}|A| = k^2|A|$

Hence, the option (B) is correct.

Question 16:

Which of the following is correct

- (A) Determinant is a square matrix.
 (B) Determinant is a number associated to a matrix.
 (C) Determinant is a number associated to a square matrix.
 (D) None of these

Answer 16:

Determinant is a number associated to a square matrix.

Hence, the option (C) is correct.

Question 1:

Find area of the triangle with vertices at the point given in each of the following:

(i) $(1, 0), (6, 0), (4, 3)$

(ii) $(2, 7), (1, 1), (10, 8)$

(iii) $(-2, -3), (3, 2), (-1, -8)$

Answer 1:

$$\text{Area of triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(i) $A(1, 0), B(6, 0), C(4, 3)$

$$\text{Area of triangle } ABC = \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [1(0 - 3) - 0(6 - 4) + 1(18 - 0)] = \frac{1}{2} (15) = 7.5 \text{ square units}$$

(ii) $A(2, 7), B(1, 1), C(10, 8)$

$$\text{Area of triangle } ABC = \frac{1}{2} \begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [2(1 - 8) - 7(1 - 10) + 1(8 - 10)] = \frac{1}{2} (47) = 23.5 \text{ square units}$$

(iii) $A(-2, -3), B(3, 2), C(-1, -8)$

$$\text{Area of triangle } ABC = \frac{1}{2} \begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [-2(2 + 8) + 3(3 + 1) + 1(-24 + 2)] = \frac{1}{2} (-30) = -15$$

Area of triangle $ABC = 15$ square units

Question 2:

Show that points $A(a, b + c), B(b, c + a), C(c, a + b)$ are collinear.

Answer 2:

If the points $A(a, b + c), B(b, c + a)$ and $C(c, a + b)$ are collinear, the area of triangle ABC will be zero.

$$\text{Area of triangle } ABC = \frac{1}{2} \begin{vmatrix} a & b + c & 1 \\ b & c + a & 1 \\ c & a + b & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} a & a + b + c & 1 \\ b & a + b + c & 1 \\ c & a + b + c & 1 \end{vmatrix} \quad [\text{Applying } C_2 \rightarrow C_1 + C_2]$$

$$= \frac{1}{2} (a + b + c) \begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} \quad [\text{Taking } a + b + c \text{ as common from } C_2]$$

$$= 0 \quad [\because C_1 = C_3]$$

Hence, the points $A(a, b + c), B(b, c + a)$ and $C(c, a + b)$ are collinear.

(Class 12)

Question 3:

Find values of k if area of triangle is 4 sq. units and vertices are

(i) $(k, 0), (4, 0), (0, 2)$

(ii) $(-2, 0), (0, 4), (0, k)$

Answer 3:

(i) $A(k, 0), B(4, 0), C(0, 2)$

$$\begin{aligned} \text{Area of triangle } ABC &= \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} \\ &= \frac{1}{2} [k(0 - 2) - 0(4 - 0) + 1(8 - 0)] = \frac{1}{2} (-2k + 8) = -k + 4 \end{aligned}$$

According to question, Area of triangle $ABC = 4$ square units

Therefore, $|-k + 4| = 4 \Rightarrow -k + 4 = \pm 4$

$\Rightarrow -k + 4 = 4$ or $-k + 4 = -4$

$\Rightarrow k = 0$ or $k = 8$

Hence, the value of k are 0 and 8.

(ii) $A(-2, 0), B(0, 4), C(0, k)$

$$\begin{aligned} \text{Area of triangle } ABC &= \frac{1}{2} \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} \\ &= \frac{1}{2} [-2(4 - k) - 0(0 - 0) + 1(0 - 0)] = \frac{1}{2} (-8 + 2k) = -4 + k \end{aligned}$$

According to question, Area of triangle $ABC = 4$ square units

Therefore, $|-4 + k| = 4 \Rightarrow -4 + k = \pm 4$

$\Rightarrow -4 + k = 4$ or $-4 + k = -4$

$\Rightarrow k = 8$ or $k = 0$

Hence, the value of k are 0 and 8.

Question 4:

(i) Find equation of line joining $(1, 2)$ and $(3, 6)$ using determinants.

(ii) Find equation of line joining $(3, 1)$ and $(9, 3)$ using determinants.

Answer 4:

(i) Let, $P(x, y)$ be any point lie on the line joining $A(1, 2)$ and $B(3, 6)$. Hence, the points A, B and P will be collinear and area of triangle ABC will be zero.

$$\text{Therefore, Area of triangle } ABP = \frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} [1(6 - y) - 2(3 - x) + 1(3y - 6x)] = 0$$

$$\Rightarrow 6 - y - 6 + 2x + 3y - 6x = 0$$

$$\Rightarrow -4x + 2y = 0$$

$$\Rightarrow 2x = y$$

(ii) Let, $P(x, y)$ be any point lie on the line joining $A(3, 1)$ and $B(9, 3)$. Hence, the points A, B and P will be collinear and area of triangle ABC will be zero.

$$\text{Therefore, Area of triangle } ABP = \frac{1}{2} \begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} [3(3 - y) - 1(9 - x) + 1(9y - 3x)] = 0$$

$$\Rightarrow 9 - 3y - 9 + x + 9y - 3x = 0$$

$$\Rightarrow -2x + 6y = 0$$

$$\Rightarrow x = 3y$$

Question 5:

If area of triangle is 35 sq. units with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$. Then k is

(A) 12 (B) -2 (C) -12, -2 (D) 12, -2

Answer 5:

$A(2, -6), B(5, 4), C(k, 4)$

$$\text{Area of triangle } ABC = \frac{1}{2} \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [2(4 - 4) + 6(5 - k) + 1(20 - 4k)] = \frac{1}{2} (30 - 6k + 20 - 4k) = 25 - 5k$$

According to question, Area of triangle $ABC = 35$ square units

$$\text{Therefore, } |25 - 5k| = 35$$

$$\Rightarrow 25 - 5k = \pm 35$$

$$\Rightarrow 25 - 5k = 35 \quad \text{or} \quad 25 - 5k = -35$$

$$\Rightarrow k = \frac{-10}{5} = -2 \quad \text{or} \quad k = \frac{60}{5} = 12$$

Hence, the option (D) is correct.

(Class 12)

Exercise 4.4

Write Minors and Cofactors of the elements of following determinants:

Question 1:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Answer 1:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

The minor of element a_{ij} is M_{ij} and the cofactor is $A_{ij} = (-1)^{i+j}M_{ij}$, therefore,

The minor of element a_{11} is $M_{11} = 3$ and the cofactor is $A_{11} = (-1)^{1+1}M_{11} = 3$

The minor of element a_{12} is $M_{12} = 0$ and the cofactor is $A_{12} = (-1)^{1+2}M_{12} = 0$

The minor of element a_{21} is $M_{21} = -4$ and the cofactor is $A_{21} = (-1)^{2+1}M_{21} = 4$

The minor of element a_{22} is $M_{22} = 2$ and the cofactor is $A_{22} = (-1)^{2+2}M_{22} = 2$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

The minor of element a_{11} is $M_{11} = d$ and the cofactor is $A_{11} = (-1)^{1+1}M_{11} = d$

The minor of element a_{12} is $M_{12} = b$ and the cofactor is $A_{12} = (-1)^{1+2}M_{12} = -b$

The minor of element a_{21} is $M_{21} = c$ and the cofactor is $A_{21} = (-1)^{2+1}M_{21} = -c$

The minor of element a_{22} is $M_{22} = a$ and the cofactor is $A_{22} = (-1)^{2+2}M_{22} = a$

Question 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Answer 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Here,

$M_{11} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0, \quad M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0, \quad M_{13} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$

$M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0, \quad M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1, \quad M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$

$M_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0, \quad M_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0, \quad M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

and $A_{ij} = (-1)^{i+j}M_{ij}$, therefore

$A_{11} = (-1)^{1+1}M_{11} = 0, \quad A_{12} = (-1)^{1+2}M_{12} = 0, \quad A_{13} = (-1)^{1+3}M_{13} = 0$

$A_{21} = (-1)^{2+1}M_{21} = 0, \quad A_{22} = (-1)^{2+2}M_{22} = 1, \quad A_{23} = (-1)^{2+3}M_{23} = 0$

$A_{31} = (-1)^{3+1}M_{31} = 0, \quad A_{32} = (-1)^{3+2}M_{32} = 0, \quad A_{33} = (-1)^{3+3}M_{33} = 1$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Here,

$$M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11, \quad M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6, \quad M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

$$M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4, \quad M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2, \quad M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20, \quad M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13, \quad M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$$

and $A_{ij} = (-1)^{i+j}M_{ij}$, therefore

$$A_{11} = (-1)^{1+1}M_{11} = 11 \quad A_{12} = (-1)^{1+2}M_{12} = -6 \quad A_{13} = (-1)^{1+3}M_{13} = 3$$

$$A_{21} = (-1)^{2+1}M_{21} = 4 \quad A_{22} = (-1)^{2+2}M_{22} = 2 \quad A_{23} = (-1)^{2+3}M_{23} = -1$$

$$A_{31} = (-1)^{3+1}M_{31} = -20 \quad A_{32} = (-1)^{3+2}M_{32} = 13 \quad A_{33} = (-1)^{3+3}M_{33} = 5$$

Question 3:

Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

Answer 3:

$$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$$

Here, $a_{21} = 2$, $a_{22} = 0$, $a_{23} = 1$ and

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = -(9 - 16) = 7$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10 - 3) = -7$$

$$\text{Therefore, } \Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 2(7) + 0(7) + 1(-7) = 7$$

Question 4:

Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

Answer 4:

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$$

Here, $a_{13} = yz$, $a_{23} = zx$, $a_{33} = xy$ and

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = -(z - x) = x - z$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$$

$$\begin{aligned}
 \text{Therefore, } \Delta &= \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = yz(z-y) + zx(x-z) + xy(y-x) \\
 &= yz^2 - y^2z + zx^2 - xz^2 + xy^2 - x^2y \\
 &= zx^2 - x^2y - xz^2 + xy^2 + yz^2 - y^2z \\
 &= x^2(z-y) - x(z^2 - y^2) + yz(z-y) \\
 &= (z-y)[x^2 - x(z+y) + yz] \\
 &= (z-y)[x^2 - xz - xy + yz] \\
 &= (z-y)[x(x-z) - y(x-z)] \\
 &= (x-z)(z-y)(x-y) \\
 &= (x-y)(y-z)(z-x)
 \end{aligned}$$

Question 5:

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ A_{ij} is Cofactors of a_{ij} , then value of Δ is given by

- (A) $a_{11}A_{11} + a_{12}A_{32} + a_{13}A_{33}$ (B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
 (C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ (D) $a_{11}A_{31} + a_{21}A_{21} + a_{31}A_{31}$

Answer 5:

The value of $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ is given by: $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Hence, the option (D) is correct.

Exercise 4.5

Find adjoint of each of the matrices in Exercises 1 and 2.

Question 1:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Answer 1:

Here, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, therefore, $A_{11} = 4$ $A_{12} = -3$ $A_{21} = -2$ $A_{22} = 1$

Adjoint of matrix $A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Question 2:

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

Answer 2:

Here, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$, therefore

$$\begin{array}{lll}
 A_{11} = 3 & A_{12} = -12 & A_{13} = 6 \\
 A_{21} = 1 & A_{22} = 5 & A_{23} = 2 \\
 A_{31} = -11 & A_{32} = -1 & A_{33} = 5
 \end{array}$$

Adjoint of matrix $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$

Verify $A(\text{adj } A) = (\text{adj } A).A = |A|.I$ in Exercises 3 and 4

Question 3:

$$\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

Answer 3:

Here, $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$, therefore,

$$A_{11} = -6 \quad A_{12} = 4 \quad A_{21} = -3 \quad A_{22} = 2$$

$$|A| = -12 + 12 = 0$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$A(\text{adj } A) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -12 + 12 & -6 + 6 \\ 24 - 24 & 12 - 12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$(\text{adj } A).A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} = \begin{bmatrix} -12 + 12 & -18 + 18 \\ 8 - 8 & 12 - 12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$|A|.I = 0 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{Hence, } A(\text{adj } A) = (\text{adj } A).A = |A|.I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Question 4:

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

Answer 4:

Here, $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, therefore, $|A| = 1(0 - 0) + 1(9 + 2) + 2(0 - 0) = 11$

$$A_{11} = 0$$

$$A_{12} = -11$$

$$A_{13} = 0$$

$$A_{21} = 3$$

$$A_{22} = 1$$

$$A_{23} = -1$$

$$A_{31} = 2$$

$$A_{32} = 8$$

$$A_{33} = 3$$

$$\text{Adjoint of matrix } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\begin{aligned} A(\text{adj } A) &= \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 0 + 11 + 0 & 3 - 1 - 2 & 2 - 8 + 6 \\ 0 + 0 + 0 & 9 + 0 + 2 & 6 + 0 - 6 \\ 0 + 0 + 0 & 3 + 0 - 3 & 2 + 0 + 9 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \end{aligned}$$

$$(\text{adj } A).A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0+0+0 & 0+2+9 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$|A|.I = 11 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$\text{Hence, } A(\text{adj } A) = (\text{adj } A).A = |A|.I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Find the inverse of each of the matrices (if it exists) given in Exercises 5 to 11.

Question 5:

$$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

Answer 5:

$$\text{Here, } A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

$$\text{Therefore, } A_{11} = 3 \quad A_{12} = -4 \quad A_{21} = 2 \quad A_{22} = 2$$

$$|A| = 6 + 8 = 14 \neq 0 \Rightarrow A^{-1} \text{ exists.}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

Question 6:

$$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

Answer 6:

$$\text{Here, } A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

$$\text{Therefore, } A_{11} = 2 \quad A_{12} = 3 \quad A_{21} = -5 \quad A_{22} = -1$$

$$|A| = -2 + 15 = 13 \neq 0 \Rightarrow A^{-1} \text{ exists.}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

Question 7:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Answer 7:

$$\text{Here, } A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\text{Therefore, } |A| = 1(10 - 0) - 2(0 - 0) + 3(0 - 0) = 10 \neq 0 \Rightarrow A^{-1} \text{ exists.}$$

$$A_{11} = 10 \quad A_{12} = 0 \quad A_{13} = 0$$

$$A_{21} = -10 \quad A_{22} = 5 \quad A_{23} = 0$$

$$A_{31} = 2 \quad A_{32} = -4 \quad A_{33} = 2$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

Question 8:

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

Answer 8:

$$\text{Here, } A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

Therefore, $|A| = 1(-3 - 0) - 0(-3 - 0) + 0(6 - 15) = -3 \neq 0 \Rightarrow A^{-1}$ exists.

$$A_{11} = -3 \qquad A_{12} = 3 \qquad A_{13} = -9$$

$$A_{21} = 0 \qquad A_{22} = -1 \qquad A_{23} = -2$$

$$A_{31} = 0 \qquad A_{32} = 0 \qquad A_{33} = 3$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{-3} \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Question 9:

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

$$\text{Here, } A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

Therefore, $|A| = 2(-1 - 0) - 1(4 - 0) + 3(8 - 7) = -3 \neq 0 \Rightarrow A^{-1}$ exists.

$$A_{11} = -1 \qquad A_{12} = -4 \qquad A_{13} = 1$$

$$A_{21} = 5 \qquad A_{22} = 23 \qquad A_{23} = -11$$

$$A_{31} = 3 \qquad A_{32} = 12 \qquad A_{33} = -6$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{-3} \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

Question 10:

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

Answer 10:

$$\text{Here, } A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

Therefore, $|A| = 1(8 - 6) + 1(0 + 9) + 2(0 - 6) = -1 \neq 0 \Rightarrow A^{-1}$ exists.

$$A_{11} = 2 \qquad A_{12} = -9 \qquad A_{13} = -6$$

$$A_{21} = 0 \qquad A_{22} = -2 \qquad A_{23} = -1$$

$$A_{31} = -1 \qquad A_{32} = 3 \qquad A_{33} = 2$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Question 11:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

Answer 11:

Here, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$, therefore

$$|A| = 1(-\cos^2 \alpha - \sin^2 \alpha) + 0(0 - 0) + 0(0 - 0) = -1 \neq 0$$

$\Rightarrow A^{-1}$ exists.

$$A_{11} = 1$$

$$A_{12} = 0$$

$$A_{13} = 0$$

$$A_{21} = 0$$

$$A_{22} = -\cos \alpha$$

$$A_{23} = -\sin \alpha$$

$$A_{31} = 0$$

$$A_{32} = -\sin \alpha$$

$$A_{33} = \cos \alpha$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

Question 12.

Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Answer 12:

Here, $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$, therefore, $A_{11} = 5$ $A_{12} = -2$ $A_{21} = -7$ $A_{22} = 3$

$$|A| = 15 - 14 = 1 \neq 0 \Rightarrow A^{-1} \text{ exists.}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$, therefore, $B_{11} = 9$ $B_{12} = -7$ $B_{21} = -8$ $B_{22} = 6$

$$|B| = 54 - 56 = -2 \neq 0 \Rightarrow B^{-1} \text{ exists.}$$

$$B^{-1} = \frac{1}{|B|} \text{adj } B = \frac{1}{|B|} \begin{bmatrix} B_{11} & B_{21} \\ B_{12} & B_{22} \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -9/2 & 4 \\ 7/2 & -3 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} -9/2 & 4 \\ 7/2 & -3 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -\frac{45}{2} - 8 & \frac{63}{2} + 12 \\ \frac{35}{2} + 6 & -\frac{49}{2} - 9 \end{bmatrix} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

$$|AB| = 67 \times 61 - 87 \times 47 = 4087 - 4089 = -2 \neq 0 \Rightarrow (AB)^{-1} \text{ exists.}$$

$$C_{11} = 61 \quad C_{12} = -47 \quad C_{21} = -87 \quad C_{22} = 67$$

$$(AB)^{-1} = \frac{1}{|AB|} \text{adj } AB = \frac{1}{|AB|} \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix}$$

Hence, $(AB)^{-1} = B^{-1}A^{-1}$ is verified.

Question 13:

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = O$. Hence, find A^{-1} .

Answer 13:

$$\begin{aligned} \text{LHS} &= A^2 - 5A + 7I = AA - 5A + 7I \\ &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O = \text{RHS} \\ &\Rightarrow A^2 - 5A + 7I = O \\ &\Rightarrow A^2 - 5A = -7I \end{aligned}$$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$$AAA^{-1} - 5AA^{-1} = -7IA^{-1}$$

$$\Rightarrow Ai - 5I = -7A^{-1} \quad [\text{Because } AA^{-1} = I]$$

$$\Rightarrow 7A^{-1} = 5I - A = 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Question 14:

For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$.

Answer 14:

Given that: $A^2 + aA + bI = O$

$$\Rightarrow \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + a \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow 4+a=0 \Rightarrow a=-4 \quad \text{and} \quad 3+a+b=0 \Rightarrow b=-3-a=-3+4=1$$

Hence, $a = -4$, $b = 1$

Question 15:

For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11I = O$. Hence,

find A^{-1} .

Answer 15:

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$$\text{LHS} = A^3 - 6A^2 + 5A + 11I$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6 \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5+11 & 7-12+5+0 & 1-6+5+0 \\ -23+18+5+0 & 27-48+10+11 & -69+84-15+0 \\ 32-42+10+0 & -13+18-5+0 & 58-84+15+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O = \text{RHS}$$

$$\Rightarrow A^3 - 6A^2 + 5A + 11I = O \quad \Rightarrow A^3 - 6A^2 + 5A = -11I$$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$$A^2AA^{-1} - 6AAA^{-1} + 5AA^{-1} = -11IA^{-1}$$

$$\Rightarrow A^2I - 6AI + 5I = -11A^{-1}$$

$$[\text{Because } AA^{-1} = I]$$

$$\Rightarrow 11A^{-1} = -A^2 + 6A - 5I$$

$$\begin{aligned} \Rightarrow 11A^{-1} &= - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 6 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \Rightarrow 11A^{-1} &= \begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix} + \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \\ \Rightarrow 11A^{-1} &= \begin{bmatrix} -4+6-5 & -2+6+0 & -1+6+0 \\ 3+6-0 & -8+12-5 & 14-18+0 \\ -7+12+0 & 3-6+0 & -14+18-5 \end{bmatrix} = \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix} \\ \Rightarrow A^{-1} &= \frac{1}{11} \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix} \end{aligned}$$

Question 16:

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1} .

Answer 16:

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$$\text{LHS} = A^3 - 6A^2 + 9A - 4I$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36+18-4 & -21+30-9+0 & 21-30+9+0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9+0 \\ 21-30+9-0 & -21+30-9-0 & 22-36+18-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O = \text{RHS}$$

$$\Rightarrow A^3 - 6A^2 + 9A - 4I = O \quad \Rightarrow A^3 - 6A^2 + 9A = 4I$$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$$A^2AA^{-1} - 6AAA^{-1} + 9AA^{-1} = 4IA^{-1}$$

$$\Rightarrow A^2I - 6AI + 9I = 4A^{-1}$$

$$[\text{Because } AA^{-1} = I]$$

$$\Rightarrow 4A^{-1} = A^2 - 6A + 9I$$

$$\begin{aligned} \Rightarrow 4A^{-1} &= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \Rightarrow 4A^{-1} &= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \\ \Rightarrow 4A^{-1} &= \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} \\ \Rightarrow A^{-1} &= \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} \end{aligned}$$

Question 17:

Let A be a non-singular square matrix of order 3×3 . Then $|\text{adj } A|$ is equal to:

- (A) $|A|$ (B) $|A|^2$ (C) $|A|^3$ (D) $3|A|$

Answer 17:

We know that $\text{adj } A = |A|I$

$$\Rightarrow (\text{adj } A)A = |A| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |(\text{adj } A)A| = |A| \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix} = |A|^3$$

$$\Rightarrow |\text{adj } A| = |A|^2,$$

Hence, the option (B) is correct.

Question 18:

If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to:

- (A) $\det(A)$ (B) $\frac{1}{\det(A)}$ (C) 1 (D) 0

Answer 18:

Given that the matrix A is invertible, hence, $A^{-1} = \frac{1}{|A|} \text{adj } A$

The order of matrix is 2, so, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Therefore, $|A| = ad - bc$ and $\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} \frac{d}{|A|} & -\frac{b}{|A|} \\ -\frac{c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$\det(A^{-1}) = |A^{-1}| = \begin{vmatrix} \frac{d}{|A|} & -\frac{b}{|A|} \\ -\frac{c}{|A|} & \frac{a}{|A|} \end{vmatrix}$$

$$= \frac{1}{|A|^2} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix} = \frac{1}{|A|^2} (ad - bc) = \frac{1}{|A|^2} |A| = \frac{1}{|A|}$$

Hence, the option (B) is correct.

Mathematics

(Chapter - 4) (Determinants)
(Class 12)
Exercise 4.6

Examine the consistency of the system of equations in Exercises 1 to 6.

Question 1:

$$x + 2y = 2$$

$$2x + 3y = 3$$

Answer 1:

The given system of equations: $x + 2y = 2$
 $2x + 3y = 3$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$|A| = 3 - 4 = -1 \neq 0 \Rightarrow A$ is non-singular and so A^{-1} exists.

Hence, the system of equations are consistent.

Question 2:

$$2x - y = 5$$

$$x + y = 4$$

Answer 2:

The given system of equations:
$$\begin{aligned} 2x - y &= 5 \\ x + y &= 4 \end{aligned}$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$|A| = 2 + 1 = 3 \neq 0 \Rightarrow A$ is non-singular and so A^{-1} exists.
Hence, the system of equations are consistent.

Question 3:

$$x + 3y = 5$$

$$2x + 6y = 8$$

Answer 3:

The given system of equations:
$$\begin{aligned} x + 3y &= 5 \\ 2x + 6y &= 8 \end{aligned}$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$|A| = 6 - 6 = 0 \Rightarrow A$ is a singular matrix and so A^{-1} does not exist. Now,

$$\text{adj } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

$$(\text{adj } A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$$

So, there is no solution of the given system of equations.

Hence, the system of equations are inconsistent.

Question 4:

$$x + y + z = 1$$

$$2x + 3y + 2z = 2$$

$$ax + ay + 2az = 4$$

Answer 4:

The given system of equations:
$$\begin{aligned} x + y + z &= 1 \\ 2x + 3y + 2z &= 2 \\ ax + ay + 2az &= 4 \end{aligned}$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$|A| = 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a) = a \neq 0$$

$\Rightarrow A$ is non-singular and so A^{-1} exists. Now,

Hence, the system of equations are consistent.

Question 5:

$$3x - y - 2z = 2$$

$$2y - z = -1$$

$$3x - 5y = 3$$

Answer 5:

$$3x - y - 2z = 2$$

The given system of equations:

$$2y - z = -1$$

$$3x - 5y = 3$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$|A| = 3(0 - 5) + 1(0 + 3) - 2(0 - 6) = -15 + 3 + 12 = 0$$

$\Rightarrow A$ is a singular matrix and so A^{-1} does not exist. Now,

$$A_{11} = -5$$

$$A_{12} = -3$$

$$A_{13} = -6$$

$$A_{21} = 10$$

$$A_{22} = 6$$

$$A_{23} = 12$$

$$A_{31} = 5$$

$$A_{32} = 3$$

$$A_{33} = 6$$

$$\text{adj } A = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$(\text{adj } A)B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 - 10 + 15 \\ -6 - 6 + 9 \\ -12 - 12 + 16 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$$

So, there is no solution of the given system of equations.

Hence, the system of equations are inconsistent.

Question 6:

$$5x - y + 4z = 5$$

$$2x + 3y + 5z = 2$$

$$5x - 2y + 6z = -1$$

Answer 6:

$$5x - y + 4z = 5$$

The given system of equations:

$$2x + 3y + 5z = 2$$

$$5x - 2y + 6z = -1$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$|A| = 5(18 + 10) + 1(12 - 25) + 4(-4 - 15) = 140 - 13 - 76 = 51 \neq 0$$

$\Rightarrow A$ is non-singular and so A^{-1} exists. Hence, the system of equations are consistent.

Solve system of linear equations, using matrix method, in Exercises 7 to 14.

Question 7:

$$5x + 2y = 4$$

$$7x + 3y = 5$$

Answer 7:

The given system of equations:

$$\begin{aligned} 5x + 2y &= 4 \\ 7x + 3y &= 5 \end{aligned}$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$|A| = 15 - 14 = 1 \neq 0 \Rightarrow A$ is non-singular and so A^{-1} exists.

Hence, the system of equations are consistent.

Now, $A_{11} = 3$ $A_{12} = -7$ $A_{21} = -2$ $A_{22} = 5$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \Rightarrow x = 2, \quad y = -3$$

Question 8:

$$2x - y = -2$$

$$3x + 4y = 3$$

Answer 8:

The given system of equations:

$$\begin{aligned} 2x - y &= -2 \\ 3x + 4y &= 3 \end{aligned}$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$|A| = 8 + 3 = 11 \neq 0 \Rightarrow A$ is non-singular and so A^{-1} exists. Now,

Hence, the system of equations are consistent.

Now, $A_{11} = 4$ $A_{12} = -3$ $A_{21} = 1$ $A_{22} = 2$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -8 + 3 \\ 6 + 6 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix} \Rightarrow x = -\frac{5}{11}, \quad y = \frac{12}{11}$$

Question 9:

$$4x - 3y = 3$$

$$3x - 5y = 7$$

Answer 9:

The given system of equations: $4x - 3y = 3$
 $3x - 5y = 7$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$|A| = -20 + 9 = -11 \neq 0 \Rightarrow A$ is non-singular and so A^{-1} exists.

Hence, the system of equations are consistent.

Now, $A_{11} = -5$ $A_{12} = -3$ $A_{21} = 3$ $A_{22} = 4$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11} \begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11} \begin{bmatrix} -15 + 21 \\ -9 + 28 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{6}{11} \\ \frac{19}{11} \end{bmatrix} \Rightarrow x = -\frac{6}{11}, \quad y = \frac{19}{11}$$

Question 10:

$$5x + 2y = 3$$

$$3x + 2y = 5$$

Answer 10:

The given system of equations: $5x + 2y = 3$
 $3x + 2y = 5$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$|A| = 10 - 6 = 4 \neq 0 \Rightarrow A$ is non-singular and so A^{-1} exists.

Hence, the system of equations are consistent.

Now, $A_{11} = 2$ $A_{12} = -3$ $A_{21} = -2$ $A_{22} = 5$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 6 - 10 \\ -9 + 25 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{4}{4} \\ \frac{16}{4} \end{bmatrix} \Rightarrow x = -1, \quad y = 4$$

Question 11:

$$2x + y + z = 1$$

$$x - 2y - z = \frac{3}{2}$$

$$3y - 5z = 9$$

Answer 11:

$$2x + y + z = 1$$

The given system of equations: $x - 2y - z = \frac{3}{2}$

$$3y - 5z = 9$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

$$|A| = 2(10 + 3) - 1(-5 - 0) + 1(3 - 0) = 26 + 5 + 3 = 34 \neq 0$$

$\Rightarrow A$ is non-singular and so A^{-1} exists. Now,

$$A_{11} = 13$$

$$A_{12} = 5$$

$$A_{13} = 3$$

$$A_{21} = 8$$

$$A_{22} = -10$$

$$A_{23} = -6$$

$$A_{31} = 1$$

$$A_{32} = 3$$

$$A_{33} = -5$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13 + 12 + 9 \\ 5 - 15 + 27 \\ 3 - 9 - 45 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -3/2 \end{bmatrix} \Rightarrow x = 1, y = \frac{1}{2}, z = -\frac{3}{2}$$

QUESTIONS 12,13 AND 14 TRY YOURSELF

Question 15:

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} . Using A^{-1} solve the system of equations

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

Answer 15:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = 0 - 6 + 5 = -1 \neq 0$$

$\Rightarrow A$ is non-singular and so A^{-1} exists. Now,

$$A_{11} = 0$$

$$A_{12} = 2$$

$$A_{13} = 1$$

$$A_{21} = -1$$

$$A_{22} = -9$$

$$A_{23} = -5$$

$$A_{31} = 2$$

$$A_{32} = 23$$

$$A_{33} = 13$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations: $2x - 3y + 5z = 11$
 $3x + 2y - 4z = -5$
 $x + y - 2z = -3$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow x = 1, y = 2, z = 3$$

Question 16:

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹ 70. Find cost of each item per kg by matrix method.

Answer 16:

Let the cost of 1 kg of onion = ₹ x ,
 Let the cost of 1 kg of wheat = ₹ y and
 Let the cost of 1 kg rice = ₹ z

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. So $4x + 3y + 2z = 60$
 The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. So, $2x + 4y + 6z = 90$ and
 The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹ 70. So, $6x + 2y + 3z = 70$

$$4x + 3y + 2z = 60$$

The given system of equations: $2x + 4y + 6z = 90$

$$6x + 2y + 3z = 70$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$|A| = 4(12 - 12) - 3(6 - 36) + 2(4 - 24) = 0 + 90 - 40 = 50 \neq 0$$

$\Rightarrow A$ is non-singular and so A^{-1} exists. Now,

$$A_{11} = 0 \quad A_{12} = 30 \quad A_{13} = -20$$

$$A_{21} = -5 \quad A_{22} = 0 \quad A_{23} = 10$$

$$A_{31} = 10 \quad A_{32} = -20 \quad A_{33} = 10$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$$\Rightarrow x = 5, y = 8, z = 8$$

Hence, the cost of 1 kg of onion is ₹ 5, the cost of 1 kg of wheat is ₹ 8 and the cost of 1 kg rice is ₹ 8.

Question 16:

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹ 70. Find cost of each item per kg by matrix method.

Answer 16:

Let the cost of 1 kg of onion = ₹ x ,

Let the cost of 1 kg of wheat = ₹ y and

Let the cost of 1 kg rice = ₹ z

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. So $4x + 3y + 2z = 60$

The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. So, $2x + 4y + 6z = 90$ and

The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹ 70. So, $6x + 2y + 3z = 70$

$$4x + 3y + 2z = 60$$

The given system of equations: $2x + 4y + 6z = 90$

$$6x + 2y + 3z = 70$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$|A| = 4(12 - 12) - 3(6 - 36) + 2(4 - 24) = 0 + 90 - 40 = 50 \neq 0$$

$\Rightarrow A$ is non-singular and so A^{-1} exists. Now,

$$A_{11} = 0$$

$$A_{12} = 30$$

$$A_{13} = -20$$

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$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$$\Rightarrow x = 5, y = 8, z = 8$$

Hence, the cost of 1 kg of onion is ₹ 5, the cost of 1 kg of wheat is ₹ 8 and the cost of 1 kg rice is ₹ 8.

ALL WORK IS TO BE DONE IN
MATHS CLASSWORK REGISTER
IT WILL BE CHECKED WHEN SCHOOL RE-OPENS.

SELF EVALUATION TEST (10 MARKS)

If you have the belief that you can do it, you will acquire all the capacity to do it even if you may not have it at the beginning! - AKS

1.

If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB .

Hence solve the following system of linear equations :

$$x - y = 3,$$

$$2x + 3y + 4z = 17 \text{ and,}$$

$$y + 2z = 7.$$

2.

Prove that $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$ is divisible by $(x + y + z)$, and hence find the quotient.

3.

Using properties of determinants, prove that $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$.

4.

Using elementary transformations, find the inverse of the matrix : $\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$.

If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, find A^{-1} .

Hence solve the following system of linear equations :

$$x - y = 3,$$

$$2x + 3y + 4z = 17 \text{ and,}$$

$$y + 2z = 7.$$

5.

OR

If a, b, c are p^{th} , q^{th} and r^{th} terms respectively of a G.P., then prove that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$.

**ALL WORK IS TO BE DONE IN
MATHS CLASSWORK REGISTER.**

Compiled by: AKS (PGT: MATHS)
ST. MARY'S PUBLIC SCHOOL
THANKS