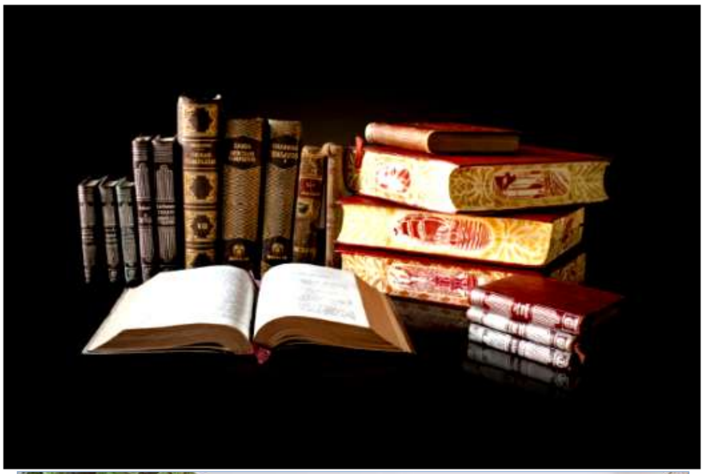
ST.MARY'S PUBLIC SCHOOL

Study Material



Note:-

1. Check the website regularly.
2. Visit relevant subject links.
3. Utilize your time well to explore, learn and share.

My dear students,

Hope you all are well. Please pay attention!

You are requested not to adjust with any short cut for your learning process. Before you start your assignment listen carefully to the links/ videos/ voice messages, we are uploading on the website as well as on the WhatsApp. If you have any doubt contact your teacher to get it cleared.

Week 3- Lesson and Assignments

FLAMINGO

L-3 DEEP WATER BY WILLIAM DOUGLAS

(https://www.youtube.com/watch?v=sFg6SlT9wzE&feature=youtu.be)

Answer the following –

Think as you read

Q. 1, 2 and 3 (page no. 27)

Q. 1,2 and 3 (page no.29)

Understanding the text

Q. 2 and 3 (page no.29)

Q. 1 (page no. 30)

Additional short answer questions:

1. Why was the YMCA pool considered safe? What did Douglas' Mother warn him about and why?
2. What was Douglas' first misadventure with water?
3. What did Douglas mean by saying "The instructor was finished, but I was not"?

POEM 3- KEEPING QUIET BY PABLO NERUDA

(https://www.youtube.com/watch?v=tvVwcY2pe7w&feature=youtu.be)

Short answer questions- Think it out

Q. 1,2,3 and 4 (page no. 96)

Reference to Context (refer Goyal's)

R.T.C. No. 1

"Perhaps the earth……. later proves to be alive.

Do all the 3 questions based on it.

R.T.C. No. 4

"Those who prepare green wars……… doing nothing.

Do all the 4 questions based on it.

R.T.C. No. 6

"It would be an exotic moment………. strangeness"

Do all the 4 questions based on it.

VISTAS

L- 3 JOURNEY TO THE END OF THE EARTH BY TISHANI DOSHI

(https://www.youtube.com/watch?v=Rj9g0d3brJM&feature=youtu.be)

Reading with insight

Q. 1,2,3and 4 (page no.23)

Additional questions

1. What are the reasons for the increasing global temperature?
2. What are the main features of the Antarctica region as discussed in the lesson?

Complete the assignments by the end of the week and keep it ready for checking.

All the Best. Stay Home Stay Safe.

Computer Sci. & I. P.

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Integrity Constraints

One of the major responsibility of a DBMS is to maintain the Integrity of the data i.e. Data being stored in the Database must be correct and valid.

An Integrity Constraints or Constraints are the rules, condition or checks applicable to a column or table which ensures the integrity or validity of data.

The following constraints are commonly used in MySQL.

- **NOT NULL**
- **PRIMARY KEY**
- **UNIQUE** *
- **DEFAULT** *
- **CHECK** *
- **FOREIGN KEY** *

Most of the constraints are applied with Column definition which are called **Column-Level (in-line Constraints)** ,but some of them may be applied at column Level as well as **Table-Level (Out-line constraints)** i.e. after defining all the columns. Ex.- Primary Key & Foreign Key

* Not included in the syllabus (recommended for advanced learning)

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Type of Constraints

S.N	Constraints	Description
1	NOT NULL	Ensures that a column cannot have NULL value.
2	DEFAULT	Provides a default value for a column, when nothing is given.
3	UNIQUE	Ensures that all values in a column are different.
4	CHECK	Ensures that all values in a column satisfy certain condition.
5	PRIMARY KEY	Used to identify a row uniquely.
6	FOREIGN KEY	Used to ensure Referential Integrity of the data.

UNIQUE v/s PRIMARY KEY

- **UNIQUE allows NULL values but PRIMERY KEY does not.**
- **Multiple column may have UNIQUE constraints, but there is only one PRIMERY KEY constraints in a table.**

Implementing Primary Key Constraints

❖Defining Primary Key at Column Level:

```
mysql> CREATE TABLE Student
       ( StCode   char(3)    NOT NULL PRIMARY KEY,
         Stname  char(20)    NOT NULL,
             ………………………..
       );
```

❖Defining Primary Key at Table Level:

```
mysql> CREATE TABLE Student
       ( StCode   char(3)    NOT NULL,
         Stname  char(20)    NOT NULL,
         ………………………..
         PRIMARY KEY (StCode) );
```

Constraint is defined after all column definitions.

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Implementing Constraints in the Table

```
mysql> CREATE TABLE Student
       (StCode char(3) NOT NULL PRIMARY KEY,
        Stname char(20)  NOT NULL,
        StAdd  varchar(40),
        AdmNo  char(5) UNIQUE,
        StSex  char(1) DEFAULT  'M',
        StAge  integer CHECK (StAge>=5) );
```

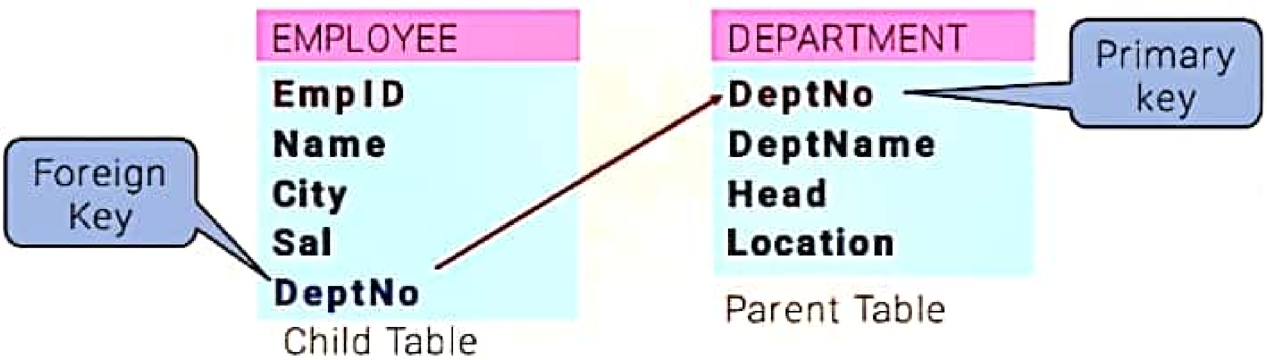
Column level constraints are defined with column definitions.

```
CREATE TABLE EMP ( Code char(3)  NOT NULL,
                   Name char(20) NOT NULL,
                   City varchar(40),
                    Pay Decimal(10,2),
                   PRIMARY KEY (Code) );
```

Table level constraints are defined after all column definitions.

Implementing Foreign Key Constraints

- A Foreign key is non-key column in a table whose value is derived from the Primary key of some other table.
- Each time when record is inserted or updated in the table, the other table is referenced. This constraints is also called Referential Integrity Constraints.
- This constraints requires two tables in which Reference table (having Primary key) called Parent table and table having Foreign key is called Child table.



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Implementing Foreign Key Cont..

Parent table

```
CREATE TABLE Department
( DeptNo     char(2)   NOT NULL PRIMARY KEY,
  DeptName   char(10)  NOT NULL,
  Head       char(30) );
```

Child Table in which Foreign key is defined.

```
CREATE TABLE Employee
( EmpNo  char(3) NOT NULL PRIMARY KEY,
  Name   char(30)  NOT NULL,
  City   char(20),
  Sal    decimal(8,2),
  DeptNo char(2),
  FOREGIN KEY (DeptNo) REFERENCES Departmet (DeptNo));
```

Parent table and column to be referenced

- A Table may have multiple Foreign keys.
- Foregn key may have repeated values i.e. Non-Key Column

Modifying Table Constraints

- Adding new column and Constraints

  ALTER TABLE <Table Name>
  ADD <Column>[<data type> <size>][<Constraints>]

```
mysql> ALTER TABLE Student ADD (TelNo Integer);
mysql> ALTER TABLE Student ADD (Age Integer CHECK (Age>=5));
mysql> ALTER TABLE Emp ADD Sal Number(8,2) DEFAULT 5000 ;
mysql> ALTER TABLE Emp ADD PRIMARY KEY (EmpID);
mysql> ALTER TABLE Emp ADD PRIMARY KEY (Name,DOB);
```

- Modifying Existing Column and Constraints

  ALTER TABLE <Table Name>
  MODIFY <Column>[<data type> <size>] [<Constraints>]

```
mysql> ALTER TABLE Student MODIFY Name VARCHAR(40);
mysql> ALTER TABLE Emp MODIFY (Sal DEFAULT 4000 );
mysql> ALTER TABLE Emp MODIFY (EmpName NOT NULL);
```

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Modifying Table Constrains cont..

- **Removing Column & Constraints**

  ALTER TABLE <Table Name>
  DROP <Column name> |<Constraints>

```
mysql> ALTER TABLE Student DROP TelNo;

mysql> ALTER TABLE Emp DROP JOB, DROP Pay;

mysql> ALTER TABLE Student DROP PRIMARY KEY;
```

- **Changing Column Name of Existing Column**

  ALTER TABLE <Table Name>
  CHANGE <Old name><New Definition>

```
mysql> ALTER TABLE Student
       CHANGE Name Stname Char(40);
```

Viewing & Disabling Constraints

- **To View the Constraints**

  The following command will show all the details like columns definitions and constraints of EMP table.

  mysql> SHOW CREATE TABLE EMP;

  Alternatively you can use **DESC**ribe command:

  mysql> DESC EMP;

- **Enabling / Disabling Foreign Key Constraint**

✓ You may enable or disable Foreign key constraints by setting the value of FOREIGN_KEY_CHECKS variable.

✓ You can't disable Primary key, however it can be dropped (deleted) by Alter Table… command.

- **To Disabling Foreign Key Constraint**

  mysql> SET FOREIGN_KEY_CHECKS = 0;

- **To Enable Foreign Key Constraint**

  mysql> SET FOREIGN_KEY_CHECKS = 1;

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Grouping Records in a Query

- Some time it is required to apply a Select query in a group of records instead of whole table.
- You can group records by using **GROUP BY <column>** clause with Select command. A group column is chosen which have non-distinct (repeating) values like City, Job etc.
- Generally, the following Aggregate Functions [MIN(), MAX(), SUM(), AVG(), COUNT()] etc. are applied on groups.

Name	Purpose
SUM()	Returns the sum of given column.
MIN()	Returns the minimum value in the given column.
MAX()	Returns the maximum value in the given column.
AVG()	Returns the Average value of the given column.
COUNT()	Returns the total number of values/ records as per given column.

Aggregate Functions & NULL Values

Consider a table Emp having following records as-

Emp		
Code	Name	Sal
E1	Ram Kumar	NULL
E2	Suchitra	4500
E3	Yogendra	NULL
E4	Sushil Kr	3500
E5	Lovely	4000

Aggregate function ignores NULL values i.e. NULL values does not play any role in calculations.

```
mysql> Select Sum(Sal) from EMP;     ⇨ 12000
mysql> Select Min(Sal) from EMP;     ⇨ 3500
mysql> Select Max(Sal) from EMP;     ⇨ 4500
mysql> Select Count(Sal) from EMP;   ⇨ 3
mysql> Select Avg(Sal) from EMP;     ⇨ 4000
mysql> Select Count(*) from EMP;     ⇨ 5
```

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Aggregate Functions & Group

An Aggregate function may applied on a column with DISTINCT or ALL keyword. If nothing is given ALL is assumed.

- **Using SUM (<Column>)**

  This function returns the sum of values in given column or expression.

```
mysql> Select Sum(Sal) from EMP;
mysql> Select Sum(DISTINCT Sal) from EMP;
mysql> Select Sum (Sal) from EMP where City='Kanpur';
mysql> Select Sum (Sal) from EMP Group By City;
mysql> Select Job, Sum(Sal) from EMP Group By Job;
```

- **Using MIN (<column>)**

This functions returns the Minimum value in the given column.

```
mysql> Select Min(Sal) from EMP;
mysql> Select Min(Sal) from EMP Group By City;
mysql> Select Job, Min(Sal) from EMP Group By Job;
```

Aggregate Functions & Group

- **Using MAX (<Column>)**

  This function returns the Maximum value in given column.

```
mysql> Select Max(Sal) from EMP;
mysql> Select Max(Sal) from EMP where City='Kanpur';
mysql> Select Max(Sal) from EMP Group By City;
```

- **Using AVG (<column>)**

This functions returns the Average value in the given column.

```
mysql> Select AVG(Sal) from EMP;
mysql> Select AVG(Sal) from EMP Group By City;
```

- **Using COUNT (<*|column>)**

This functions returns the number of rows in the given column.

```
mysql> Select Count (*) from EMP;
mysql> Select Count(Sal) from EMP Group By City;
mysql> Select Count(*), Sum(Sal) from EMP Group By Job;
```

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Aggregate Functions & Conditions

You may use any condition on group, if required. HAVING <condition> clause is used to apply a condition on a group.

```
mysql> Select Job, Sum(Pay) from EMP
              Group By Job HAVING Sum(Pay)>=8000;
mysql> Select Job, Sum(Pay) from EMP
              Group By Job HAVING Avg(Pay)>=7000;
mysql> Select Job, Sum(Pay) from EMP
              Group By Job HAVING Count(*)>=5;
mysql> Select Job, Min(Pay),Max(Pay), Avg(Pay) from EMP
              Group By Job HAVING Sum(Pay)>=8000;
mysql> Select Job, Sum(Pay) from EMP Where City='Dehradun'
              Group By Job HAVING Count(*)>=5;
```

'Having' is used with Group By Clause only.

Where clause works in respect of whole table but Having works on Group only. If Where and Having both are used then Where will be executed first.

Displaying Data from Multiple Tables - Join Query

Some times it is required to access the information from two or more tables, which requires the Joining of two or more tables. Such query is called Join Query.

MySQL facilitates you to handle Join Queries. The major types of Join is as follows-

- **Cross Join (Cartesian Product)**
- **Equi Join**
- **Non-Equi Join**
- **Natural Join**

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Cross Join – Mathematical Principle

Consider the two set **A= {a,b}** and **B={1,2}**

The Cartesian Product i.e. **AxB = {(a,1) (a,2) (b,1) (b,2)}**

Similarly, we may compute Cross Join of two tables by joining each Record of first table with each record of second table.

R

A	B	C
p	q	s
m	n	t
o	p	s
l	m	u

X

S

C	X	Y
s	q	r
t	n	m
o	p	s

R x S

A	B	C	C	X	Y
p	q	s	s	q	r
p	q	s	t	n	m
p	q	s	o	p	s
m	n	t	s	q	r
m	n	t	t	n	m
m	n	t	o	p	s
...	...	...	...	...	...
l	m	u	o	p	s

The table will contain (4x3=12) rows and 6 columns.

Equi Join – Mathematical Principle

In Equvi Join, records are joined on the equality condition of Joining Column. Generally, the Join column is a column which is common in both tables.

Consider the following table **R** and **S** having **C** as Join column.

R

A	B	C
p	q	s
m	n	t
o	p	s
l	m	u

S

C	X	Y
s	q	r
t	n	m
o	p	s

T (Equi Join)

A	B	C	C	X	Y
p	q	s	s	q	r
m	n	t	t	n	m
o	p	s	s	p	r

The result table will contain 6 columns but records are selected those are having Equal value for **C** column in both table.

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Non-Equi Join – Mathematical Principle

In Non-Equi Join, records are joined on the condition other than Equal operator (>,<,<>,>=,<=) for Joining Column (common column).

Consider the following table **R** and **S** having **C** as Join column and **< >** (not equal) operator is applied in join condition.

R

A	B	C
p	q	s
m	n	t
o	p	s
l	m	u

< >

S

C	X	Y
s	q	r
t	n	m
o	p	s

T (Non-Equi Join)

A	B	C	C	X	Y
p	q	s	t	n	m
p	q	s	o	p	s
m	n	t	s	p	r
m	n	t	o	p	s
o	p	s	t	n	m
o	p	s	o	p	s
l	m	u	s	q	R
l	m	u	t	n	M
l	m	u	o	p	s

The result table will contain 6 columns but records are selected those are having not- equal value for **C** column in both table.

Natural Join – Mathematical Principle

The Natural Join is much similar to Equi Join i.e. records are joined on the equality condition of Joining Column except that the common column appears one time.

Consider the following table **R** and **S** having **C** as Join column.

R

A	B	C
p	q	s
m	n	t
o	p	s
l	m	u

S

C	X	Y
s	q	r
t	n	m
o	p	s

T (Natural Join)

A	B	C	X	Y
p	q	s	q	r
m	n	t	n	m
o	p	s	p	r

The result table will contain **5** columns (common column is eliminated) but records are selected those are having Equal value for C column in both table.

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Implementing Join Operation in MySQL

Consider the two tables EMP and DEPT -

Primary Key

Foreign Key

EMP

EmpID	EName	City	Job	Pay	DeptNo
E1	Amitabh	Mumbai	Manager	50000	D1
E2	Sharukh	Delhi	Manager	40000	D2
E3	Amir	Mumbai	Engineer	30000	D1
E4	Kimmi	Kanpur	Operator	10000	D2
E4	Puneet	Chennai	Executive	18000	D3
E5	Anupam	Kolkatta	Manager	35000	D3
E6	Syna	Banglore	Secretary	15000	D1
...	...	...	...	...	...

DEPT

Primary Key

DeptNo	DName	Location
D1	Production	Mumbai
D2	Sales	Delhi
D3	Admn	Mumbai
D4	Research	Chennai

Suppose we want complete details of employees with their Deptt. Name and Location…… this query requires the join of both tables

How to Join ?

MySQL offers different ways by which you may join two or more tables.

- **Method 1 : Using Multiple table with FROM clause**

  The simplest way to implement JOIN operation, is the use of multiple table with FROM clause followed with Joining condition in WHERE clause.

```
Select * From EMP, DEPT
Where Emp.DeptNo = Dept.DeptNo ;
```

To avoid ambiguity you should use Qualified name i.e. <Table>.<column>

If common column are differently spelled then no need to use Qualified name.

- **Method 2: Using JOIN keyword**

  MySQL offers JOIN keyword, which can be used to implement all type of Join operation.

```
Select * From EMP JOIN DEPT ON Emp.DeptNo=Dept.DeptNo ;
```

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Using Multiple Table with FROM clause

The General Syntax of Joining table is-

SELECT < List of Columns> FROM <Table1, Table 2, ... >
WHERE <Joining Condition> [Order By ..] [Group By ..]

- You may add more conditions using AND/OR NOT operators, if required.
- All types of Join (Equi, No-Equi, Natural etc. are implemented by <u>changing the Operators in Joining Condition and selection of columns</u> with SELECT clause.

Ex. Find out the name of Employees working in Production Deptt.

```
Select  Ename From EMP, DEPT
Where Emp.DeptNo=Dept.DeptNo AND Dname='Production';
```

Ex. Find out the name of Employees working in same city from where they belongs (hometown).

```
Select  Ename From EMP, DEPT
Where Emp.DeptNo=Dept.DeptNo And City=Location;
```

Using JOIN keyword with FROM clause

MySQL 's JOIN Keyword may be used with From clause.

SELECT < List of Columns>
FROM <Table1> JOIN <Table2> ON <Joining Condition>
[WHERE <Condition>] [Order By ..] [Group By ..]

Ex. Find out the name of Employees working in Production Deptt.

```
Select  Ename From EMP JOIN DEPT ON Emp.DeptNo=Dept.DeptNo
Where Dname='Production';
```

Ex. Find out the name of Employees working in same city from where they belongs (hometown) .

```
Select  Ename From EMP JOIN DEPT ON Emp.DeptNo = Dept.DeptNo
WHERE City=Location;
```

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Nested Query (A query within another query)

Sometimes it is required to join two sub-queries to solve a problem related to the single or multiple table. Nested query contains multiple query in which inner query evaluated first.

The general form to write Nested query is-

```
Select …. From <Table>
Where <Column1> <Operator>
(Select Column1 From <Table> [Where <Condition>])
```

Ex. Find out the name of Employees working in Production Deptt.

```
Select  Ename From EMP
 Where DeptNo = (Select DeptNo From DEPT Where
                          DName='Production');
```

Ex. Find out the name of Employees who are getting more pay than 'Ankit'.

```
Select  Ename From EMP
Where Pay >= (Select Pay From EMP Where Ename='Ankit' );
```

Union of Tables

Sometimes it is required to combine all records of two tables without having duplicate records. The combining records of two tables is called UNION of tables.

UNION Operation is similar to UNION of Set Theory.

E.g. If set A= {a,c,m,p,q} and Set B= {b,m,q,t,s}

Then AUB= {a,c,m,p,q,b,t,s}

[All members of Set A and Set B are taken without repeating]

```
Select …. From <Table1>[Where <Condition>]
UNION [ALL]
Select …. From <Table2> [Where <Condition>];
```

Ex.
```
Select  Ename From PROJECT1
UNION
Select  Ename From PROJECT2 ;
```

Both tables or output of queries must be UNION compatible i.e. they must be same in column structure (number of columns and data types must be same).

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MAGNETIC EFFECT OF CURRENT - I

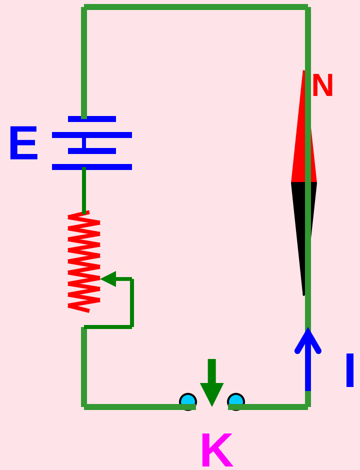
1. Magnetic Effect of Current – Oersted's Experiment
2. Ampere's Swimming Rule
3. Maxwell's Cork Screw Rule
4. Right Hand Thumb Rule
5. Biot – Savart's Law
6. Magnetic Field due to Infinitely Long Straight Current – carrying Conductor
7. Magnetic Field due to a Circular Loop carrying current
8. Magnetic Field due to a Solenoid

Magnetic Effect of Current:

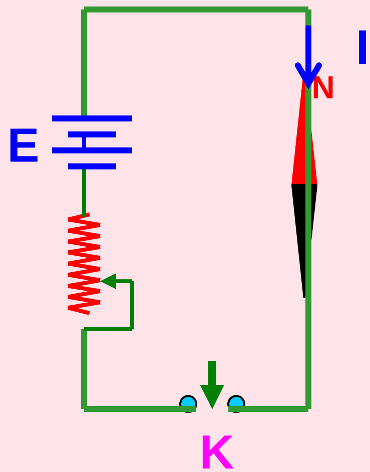
An electric current (i.e. flow of electric charge) produces magnetic effect in the space around the conductor called strength of Magnetic field or simply Magnetic field.

Oersted's Experiment:

When current was allowed to flow through a wire placed parallel to the axis of a magnetic needle kept directly below the wire, the needle was found to deflect from its normal position.



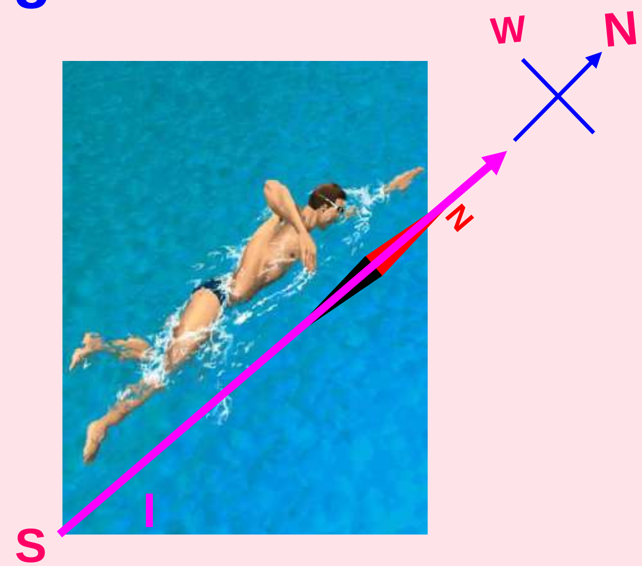
When current was reversed through the wire, the needle was found to deflect in the opposite direction to the earlier case.



Rules to determine the direction of magnetic field:

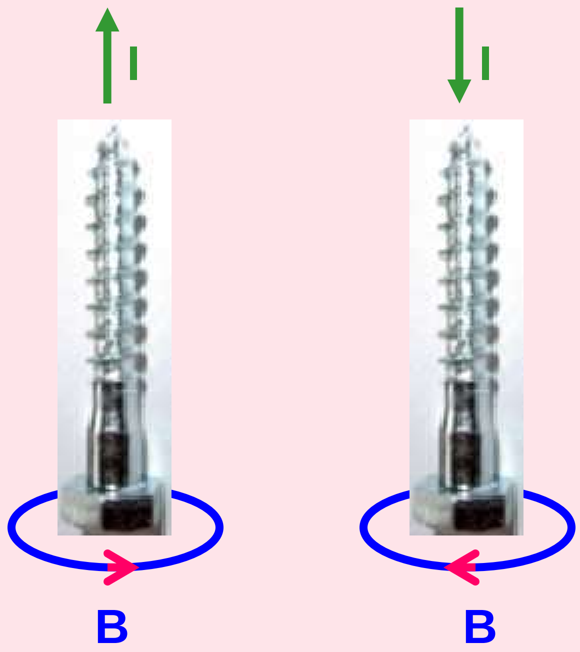
Ampere's Swimming Rule:

Imagining a man who swims in the direction of current from south to north facing a magnetic needle kept under him such that current enters his feet then the North pole of the needle will deflect towards his left hand, i.e. towards West.



Maxwell's Cork Screw Rule or Right Hand Screw Rule:

If the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.

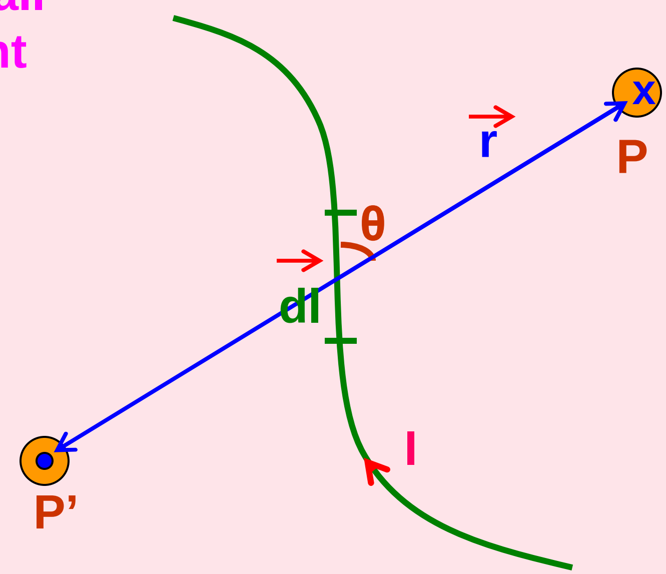


Right Hand Thumb Rule or Curl Rule:

If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the fingers encircling the conductor will give the direction of the magnetic lines of force.

Biot – Savart's Law:

The strength of magnetic field dB due to a small current element dl carrying a current I at a point P distant r from the element is directly proportional to I, dl, sin θ and inversely proportional to the square of the distance (r^2) where θ is the angle between dl and r.



i) $dB \propto I$

ii) $dB \propto dl$

iii) $dB \propto \sin\theta$

iv) $dB \propto 1 / r^2$

$$dB \propto \frac{I\, dl \sin\theta}{r^2}$$

$$\boxed{dB = \frac{\mu_0}{4\pi}\frac{I\, dl \sin\theta}{r^2}}$$

Biot – Savart's Law in vector form:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r}}{r^3}}$$

Value of $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1} or Wb m^{-1} A^{-1}

Direction of $d\vec{B}$ is same as that of direction of $d\vec{l} \times \vec{r}$ which can be determined by Right Hand Screw Rule.

It is emerging $\odot$ at P' and entering $\otimes$ at P into the plane of the diagram.

Current element is a vector quantity whose magnitude is the vector product of current and length of small element having the direction of the flow of current. ($I\, d\vec{l}$)

Magnetic Field due to a Straight Wire carrying current:

According to Biot – Savart's law

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}$$

$\sin\theta = a / r \;\; = \cos\Phi$

or $\quad r = a / \cos\Phi$

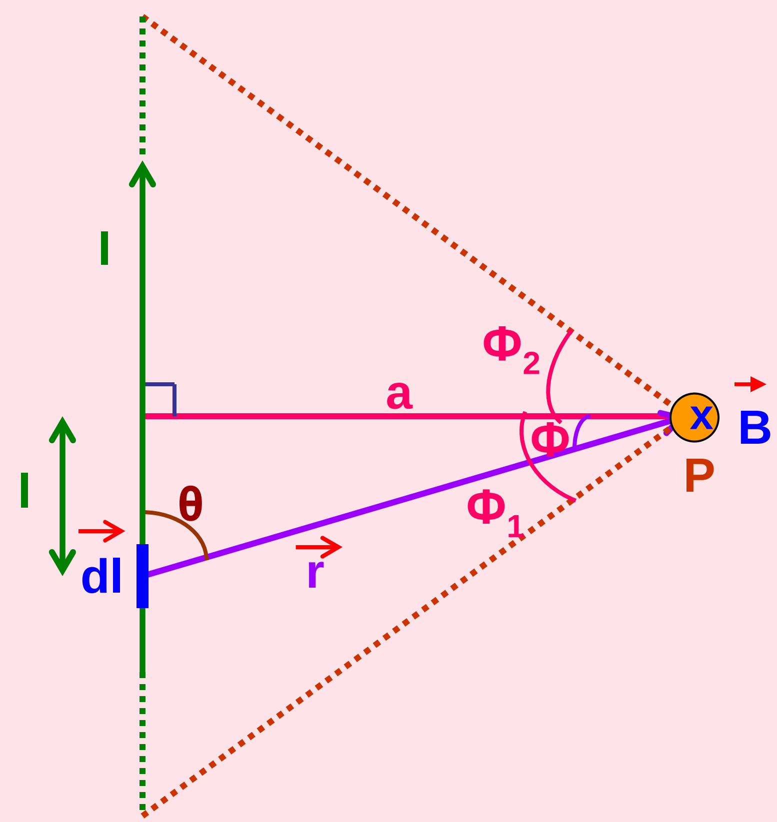
$\tan\Phi = l / a$

or $\quad l = a \tan\Phi$

$dl = a \sec^2\Phi \, d\Phi$

Substituting for r and dl in dB,

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{I \cos\Phi \, d\Phi}{a}}$$



Magnetic field due to whole conductor is obtained by integrating with limits $-\Phi_1$ to Φ_2. (Φ_1 is taken negative since it is anticlockwise)

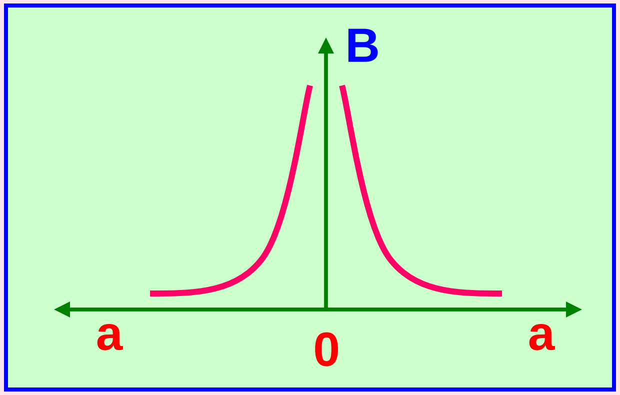
$$B = \int dB = \int_{-\Phi_1}^{\Phi_2} \frac{\mu_0}{4\pi} \frac{I \cos\Phi \, d\Phi}{a}$$

$$\boxed{B = \frac{\mu_0 I (\sin\Phi_1 + \sin\Phi_2)}{4\pi a}}$$

If the straight wire is infinitely long,

then $\Phi_1 = \Phi_2 = \pi / 2$

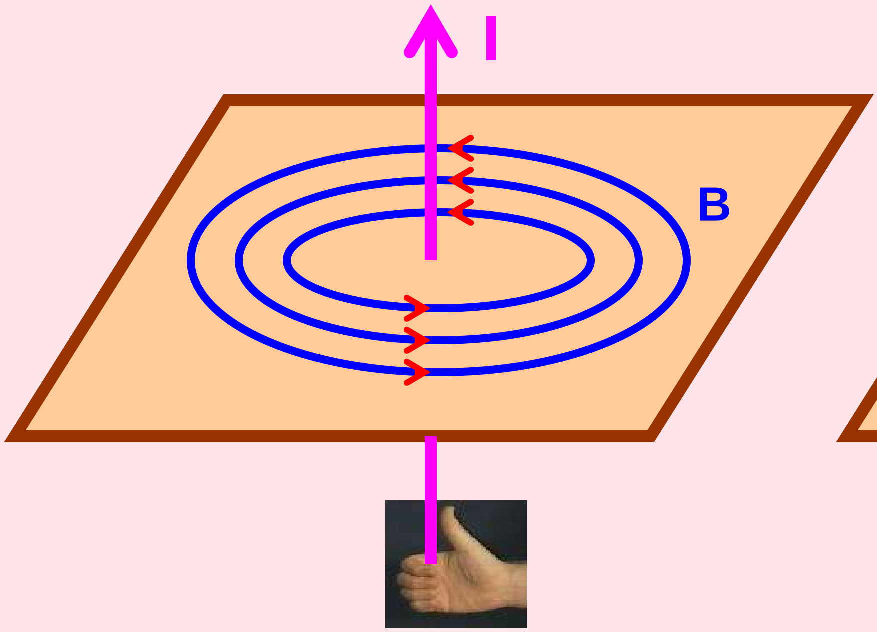
$$B = \frac{\mu_0 \, 2I}{4\pi a} \quad \text{or} \quad B = \frac{\mu_0 \, I}{2\pi a}$$

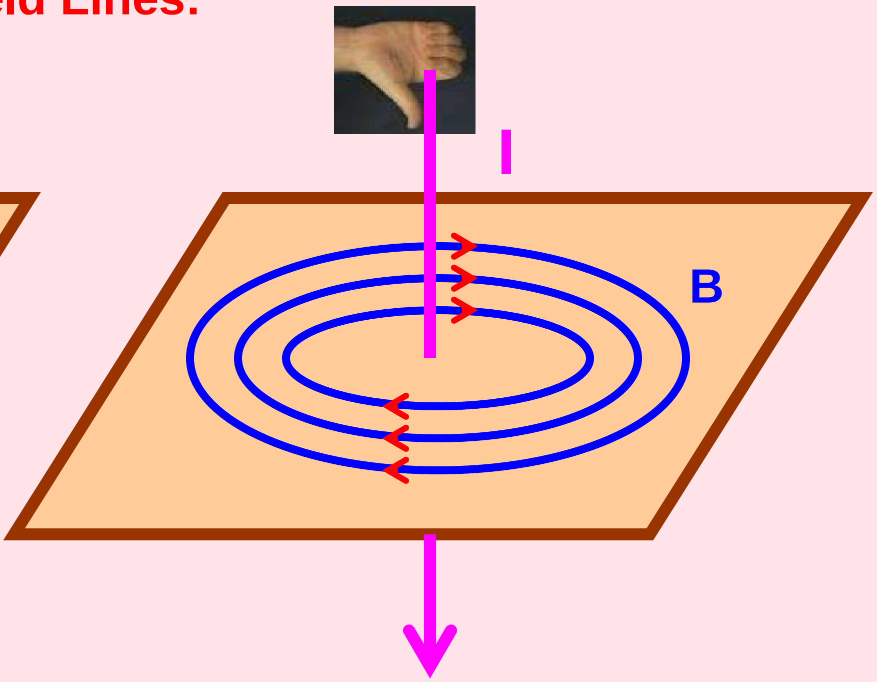


Direction of $\vec{B}$ is same as that of direction of $\vec{dl} \times \vec{r}$ which can be determined by Right Hand Screw Rule.

It is perpendicular to the plane of the diagram and entering into the plane at P.

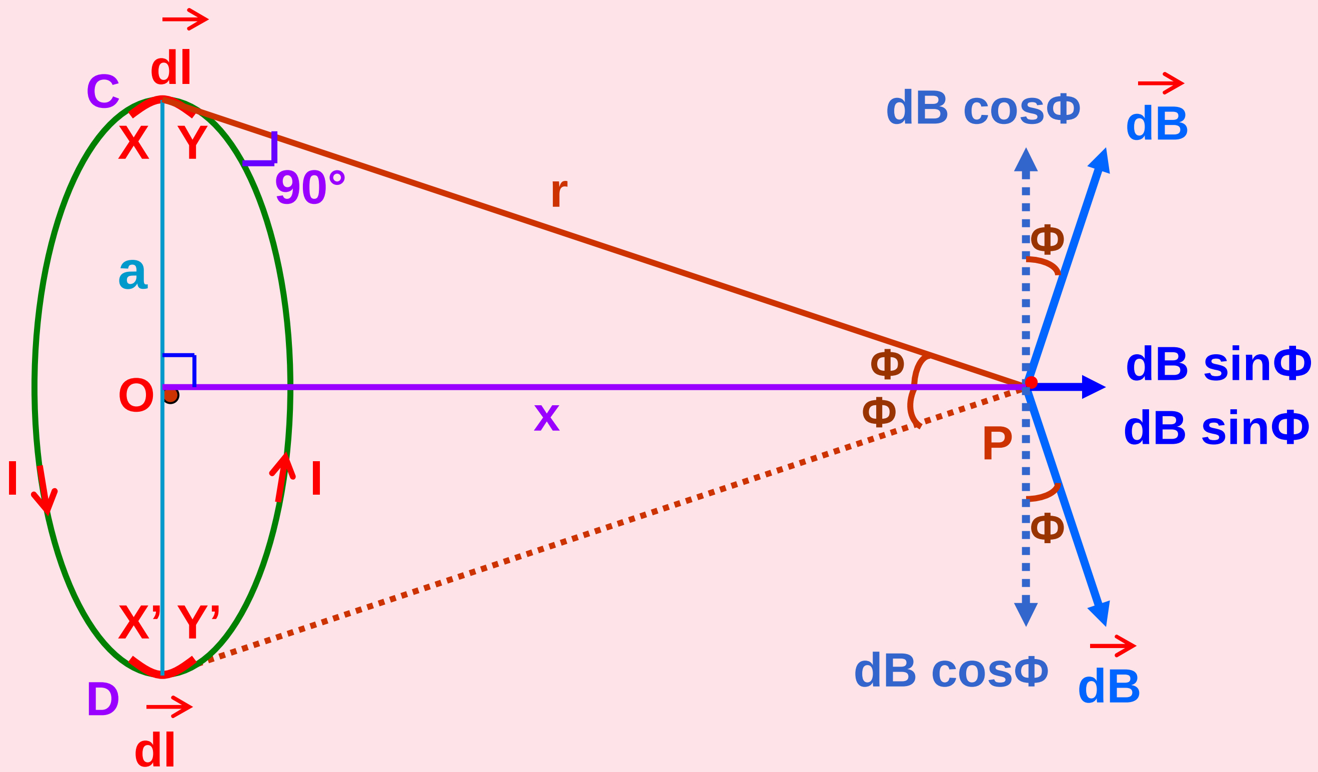
Magnetic Field Lines:





Magnetic Field due to a Circular Loop carrying current:

1) At a point on the axial line:



The plane of the coil is considered perpendicular to the plane of the diagram such that the direction of magnetic field can be visualized on the plane of the diagram.

At C and D current elements XY and X'Y' are considered such that current at C emerges out and at D enters into the plane of the diagram.

$$dB = \frac{\mu_0 \; I \, dl \sin\theta}{4\pi \quad r^2} \qquad \text{or} \qquad dB = \frac{\mu_0 \; I \, dl}{4\pi \; r^2}$$

The angle θ between $\vec{dl}$ and $\vec{r}$ is 90° because the radius of the loop is very small and since sin 90° = 1

The semi-vertical angle made by $\vec{r}$ to the loop is Φ and the angle between $\vec{r}$ and $\vec{dB}$ is 90° . Therefore, the angle between vertical axis and $\vec{dB}$ is also Φ.

$\vec{dB}$ is resolved into components dB cosΦ and dB sinΦ .

Due to diametrically opposite current elements, cosΦ components are always opposite to each other and hence they cancel out each other.

SinΦ components due to all current elements $\vec{dl}$ get added up along the same direction (in the direction away from the loop).

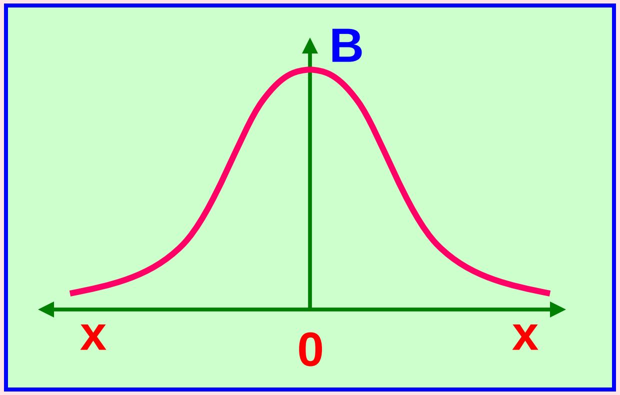
$$B = \int dB \sin\Phi = \int \frac{\mu_0 \; I \, dl \sin\Phi}{4\pi \; r^2} \qquad \text{or} \qquad B = \frac{\mu_0 \, I \, (2\pi a) \, a}{4\pi \, (a^2 + x^2) \, (a^2 + x^2)^{1/2}}$$

$$\boxed{B = \frac{\mu_0 \, I \, a^2}{2(a^2 + x^2)^{3/2}}}$$

(μ_0 , I, a, sinΦ are constants, $\int dl = 2\pi a$ and r & sinΦ are replaced with measurable and constant values.)

Special Cases:

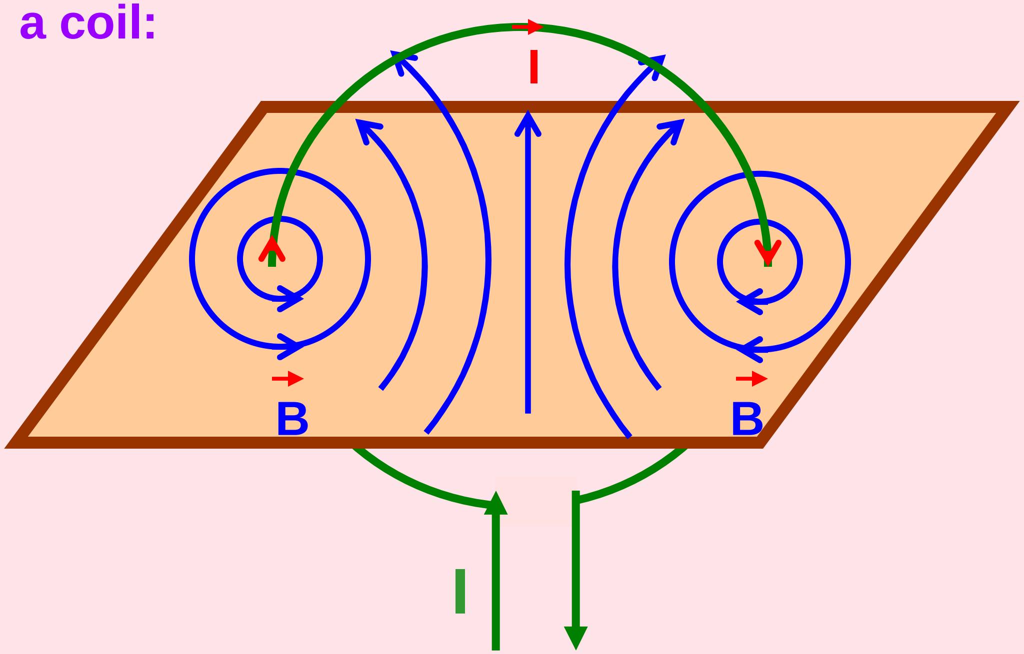
i) At the centre O, x = 0. $\therefore$ $B = \dfrac{\mu_0 I}{2a}$

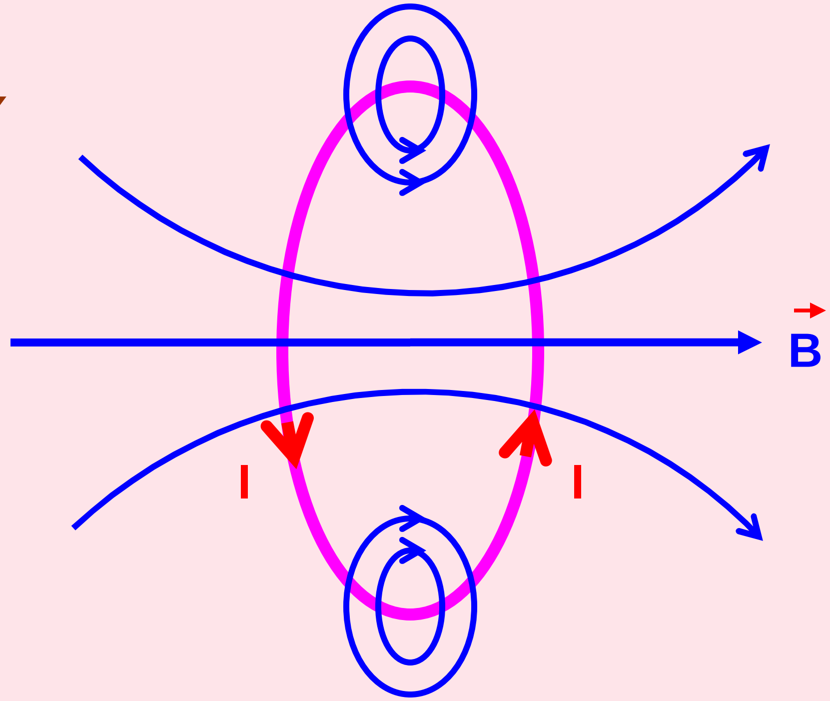


ii) If the observation point is far away from the coil, then a << x. So, a^2 can be neglected in comparison with x^2.

$\therefore$ $B = \dfrac{\mu_0 I a^2}{2x^3}$

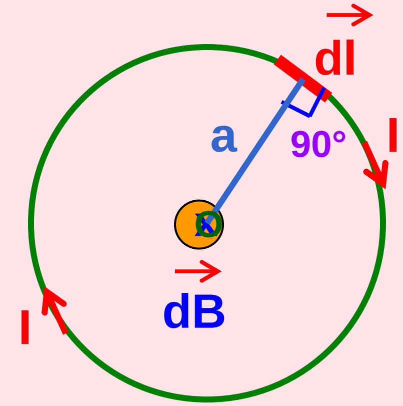
Different views of direction of current and magnetic field due to circular loop of a coil:





2) B at the centre of the loop:

The plane of the coil is lying on the plane of the diagram and the direction of current is clockwise such that the direction of magnetic field is perpendicular and into the plane.



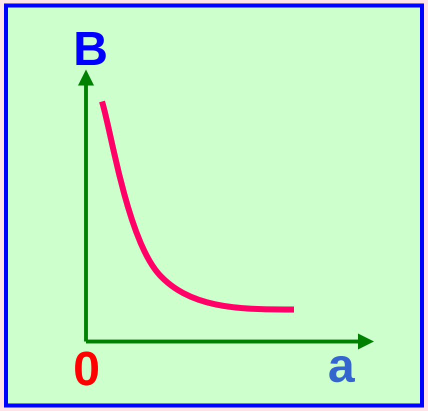
$$dB = \frac{\mu_0 \; I \; dl \sin\theta}{4\pi \; a^2}$$

$$dB = \frac{\mu_0 \; I \; dl}{4\pi \; a^2}$$

The angle θ between dl and a is 90° because the radius of the loop is very small and since sin 90°= 1

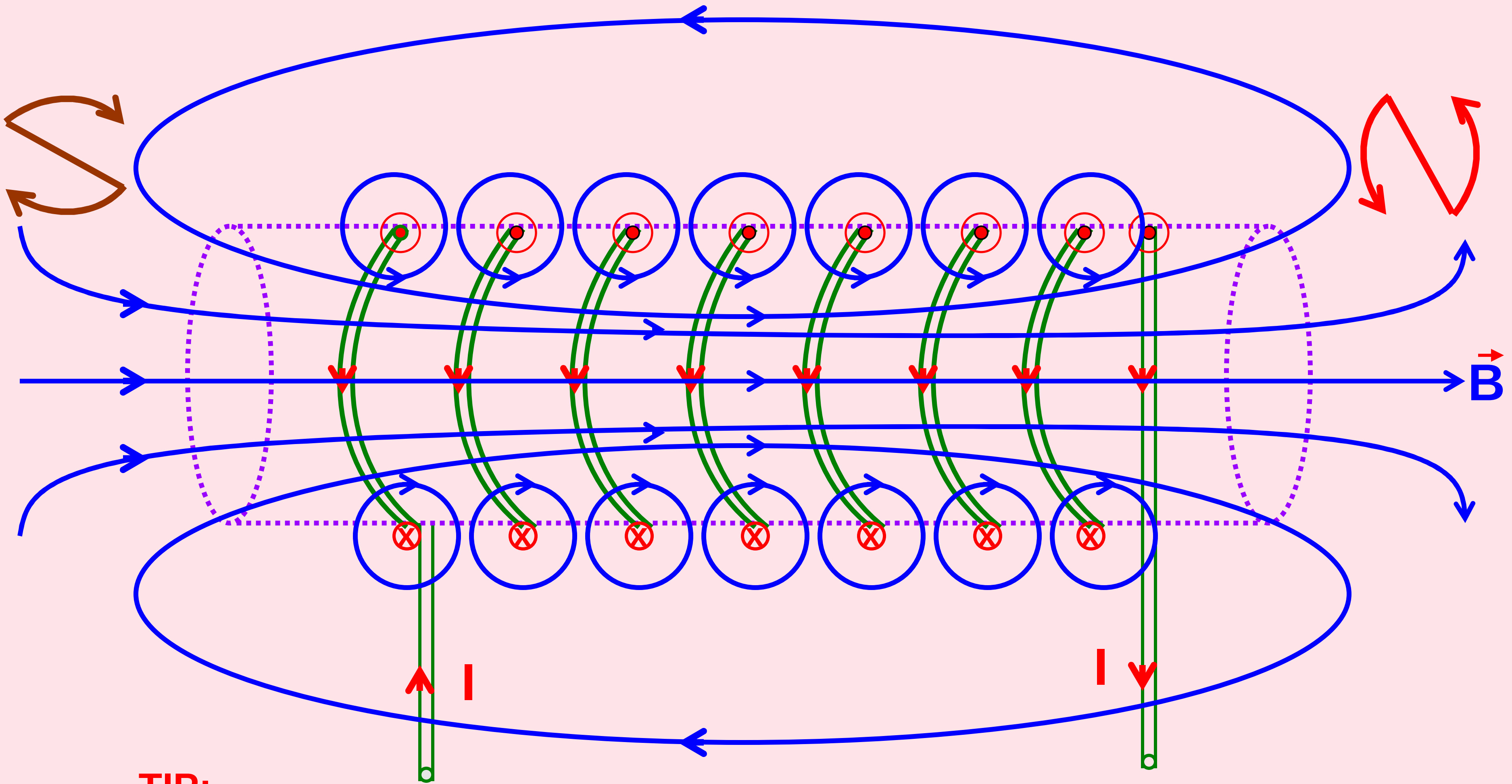
$$B = \int dB = \int \frac{\mu_0 \; I \; dl}{4\pi \; a^2}$$

$$\boxed{B = \frac{\mu_0 I}{2a}}$$



(μ_0 , I, a are constants and $\int dl = 2\pi a$)

Magnetic Field due to a Solenoid:



TIP:

When we look at any end of the coil carrying current, if the current is in anti-clockwise direction then that end of coil behaves like North Pole and if the current is in clockwise direction then that end of the coil behaves like South Pole.

MAGNETIC EFFECT OF CURRENT - II

1. Lorentz Magnetic Force
2. Fleming's Left Hand Rule
3. Force on a moving charge in uniform Electric and Magnetic fields
4. Force on a current carrying conductor in a uniform Magnetic Field
5. Force between two infinitely long parallel current-carrying conductors
6. Definition of ampere
7. Representation of fields due to parallel currents
8. Torque experienced by a current-carrying coil in a uniform Magnetic Field
9. Moving Coil Galvanometer
10. Conversion of Galvanometer into Ammeter and Voltmeter
11. Differences between Ammeter and Voltmeter

Lorentz Magnetic Force:

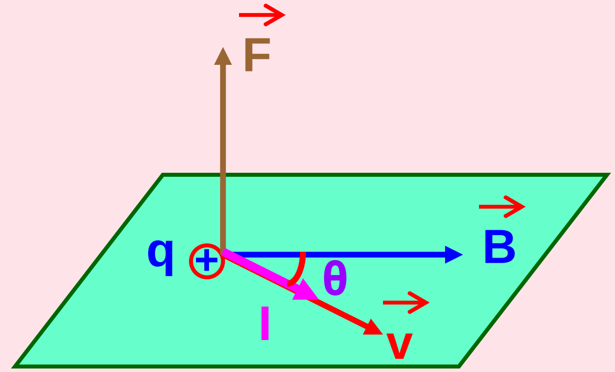
A current carrying conductor placed in a magnetic field experiences a force which means that a moving charge in a magnetic field experiences force.

$$\vec{F}_m = q\,(\vec{v} \times \vec{B})$$

or

$$\vec{F}_m = (q\,v\,B\sin\theta)\,\hat{n}$$

where θ is the angle between $\vec{v}$ and $\vec{B}$



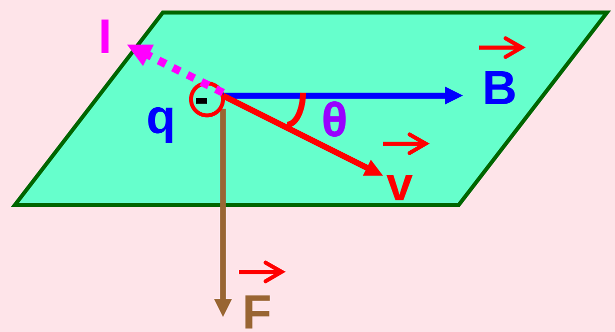
Special Cases:

i) If the charge is at rest, i.e. v = 0, then F_m = 0. So, a stationary charge in a magnetic field does not experience any force.

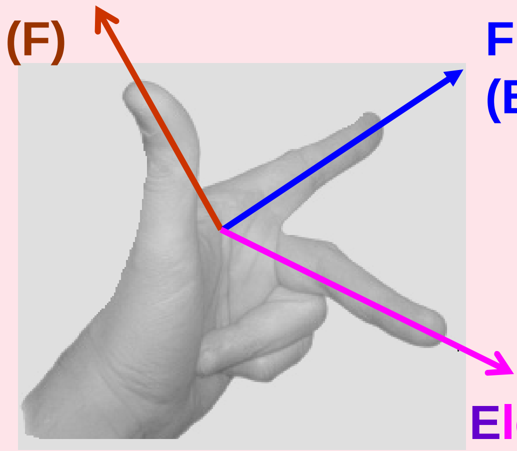
ii) If θ = 0° or 180° i.e. if the charge moves parallel or anti-parallel to the direction of the magnetic field, then F_m = 0.

iii) If θ = 90° i.e. if the charge moves perpendicular to the magnetic field, then the force is maximum.

$$F_{m\,(max)} = q\,v\,B$$



Fleming's Left Hand Rule:



If the central finger, fore finger and thumb of left hand are stretched mutually perpendicular to each other and the central finger points to current, fore finger points to magnetic field, then thumb points in the direction of motion (force) on the current carrying conductor.

TIP:

Remember the phrase 'e m f' to represent electric current, magnetic field and force in anticlockwise direction of the fingers of left hand.

Force on a moving charge in uniform Electric and Magnetic Fields:

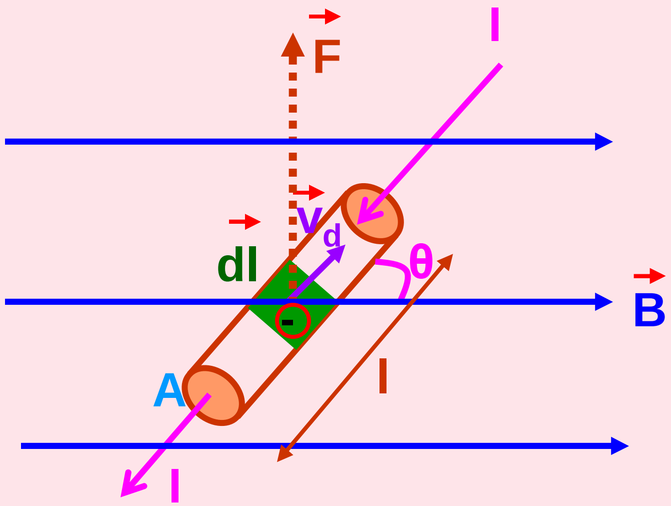
When a charge q moves with velocity $\vec{v}$ in region in which both electric field $\vec{E}$ and magnetic field $\vec{B}$ exist, then the Lorentz force is

$\vec{F} = q\vec{E} + q\,(\vec{v} \times \vec{B})$ or $\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$

Force on a current-carrying conductor in a uniform Magnetic Field:

Force experienced by each electron in the conductor is

$\vec{f} = -e(\vec{v}_d \times \vec{B})$



If n be the number density of electrons, A be the area of cross section of the conductor, then no. of electrons in the element dl is n A dl.

Force experienced by the electrons in dl is

$\vec{dF} = n\,A\,dl\,[-e(\vec{v}_d \times \vec{B})] = -\,n\,e\,A\,v_d\,(\vec{dl} \times \vec{B})$

$= I\,(\vec{dl} \times \vec{B})$

where $I = neAv_d$ and -ve sign represents that the direction of dl is opposite to that of v_d)

$\vec{F} = \int \vec{dF} = \int I\,(\vec{dl} \times \vec{B})$

$\boxed{\vec{F} = I\,(\vec{l} \times \vec{B})}$ or $\boxed{F = I\,l\,B \sin\theta}$

Forces between two parallel infinitely long current-carrying conductors:

Magnetic Field on RS due to current in PQ is

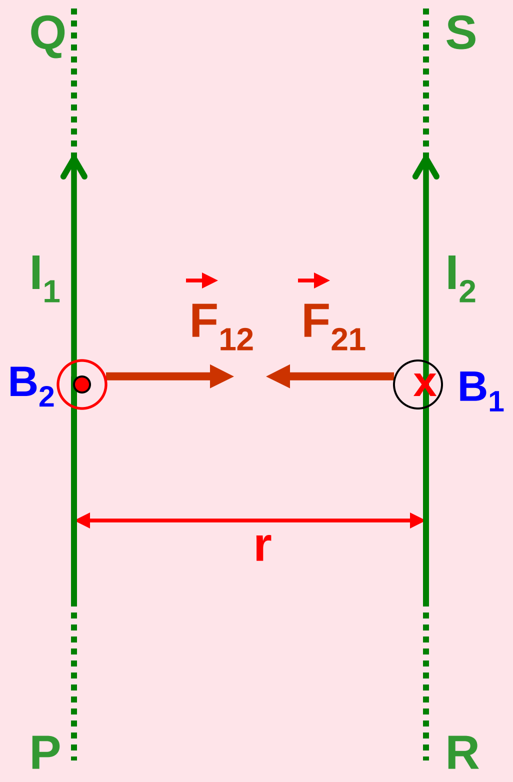
$$B_1 = \frac{\mu_0 I_1}{2\pi r} \quad \text{(in magnitude)}$$

Force acting on RS due to current I_2 through it is

$$F_{21} = \frac{\mu_0 I_1}{2\pi r} I_2 \, l \sin 90^\circ \quad \text{or} \quad F_{21} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

B_1 acts perpendicular and into the plane of the diagram by Right Hand Thumb Rule. So, the angle between l and B_1 is 90° .

l is length of the conductor.



Magnetic Field on PQ due to current in RS is

$$B_2 = \frac{\mu_0 I_2}{2\pi r} \quad \text{(in magnitude)}$$

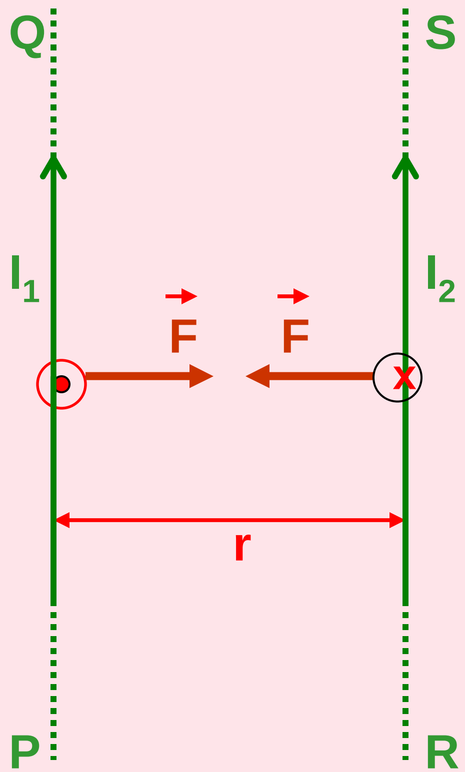
Force acting on PQ due to current I_1 through it is

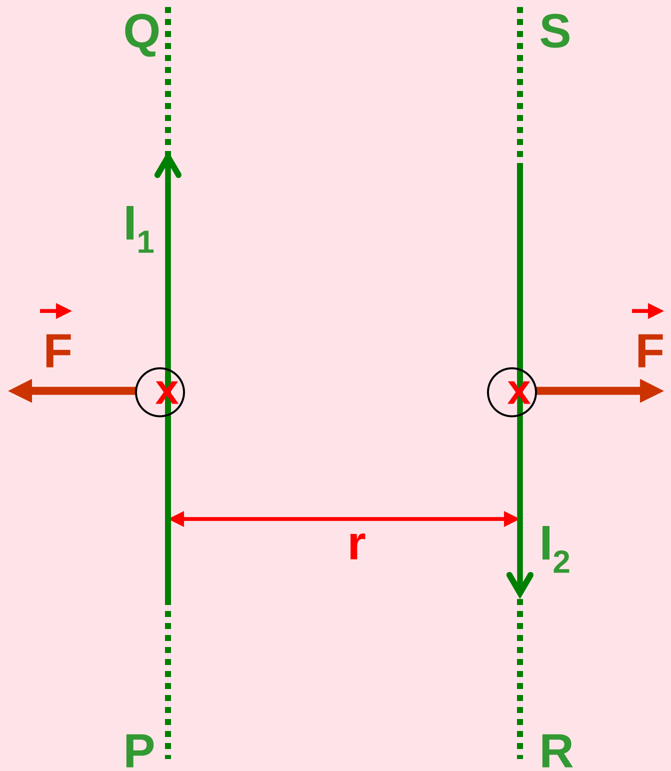
$$F_{12} = \frac{\mu_0 I_2}{2\pi r} I_1 \, l \sin 90^\circ \quad \text{or} \quad F_{12} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

(The angle between l and B_2 is 90° and B_2 Is emerging out)

$$\boxed{F_{12} = F_{21} = F = \frac{\mu_0 I_1 I_2 l}{2\pi r}}$$

Force per unit length of the conductor is $\boxed{F / l = \frac{\mu_0 I_1 I_2}{2\pi r}}$ N / m





By Fleming's Left Hand Rule, the conductors experience force towards each other and hence attract each other.

By Fleming's Left Hand Rule, the conductors experience force away from each other and hence repel each other.

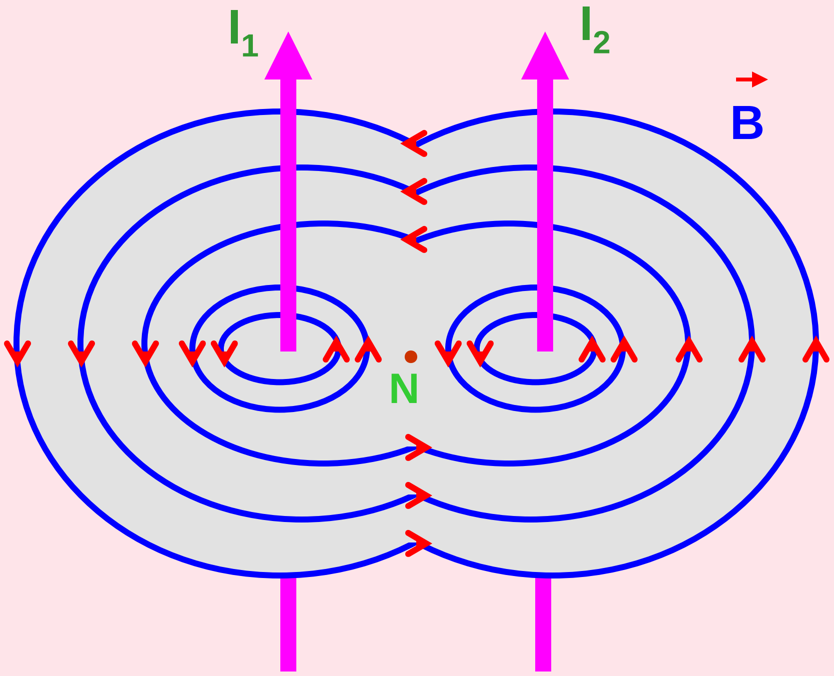
Definition of Ampere:

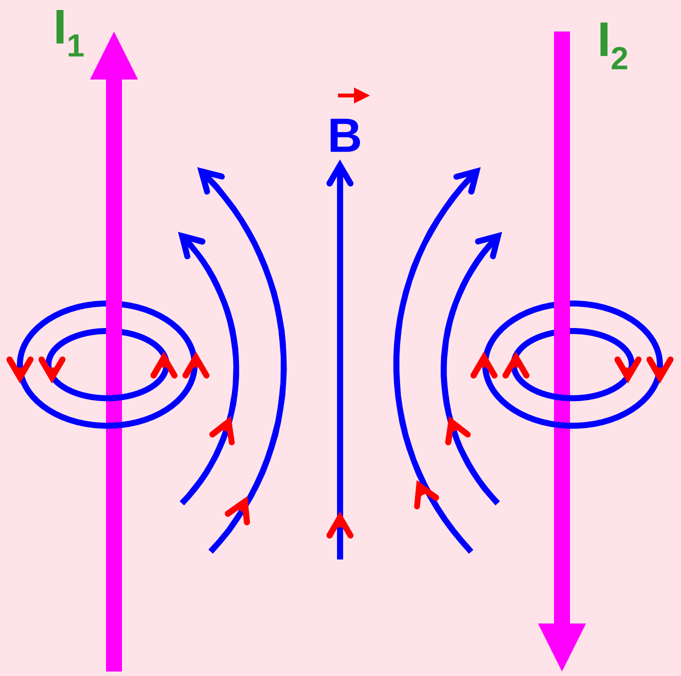
Force per unit length of the conductor is $F / l = \dfrac{\mu_0 \, I_1 \, I_2}{2\pi \, r}$ N / m

When $I_1 = I_2 = 1$ Ampere and $r = 1$ m, then $F = 2 \times 10^{-7}$ N/m.

One ampere is that current which, if passed in each of two parallel conductors of infinite length and placed 1 m apart in vacuum causes each conductor to experience a force of 2×10^{-7} Newton per metre of length of the conductor.

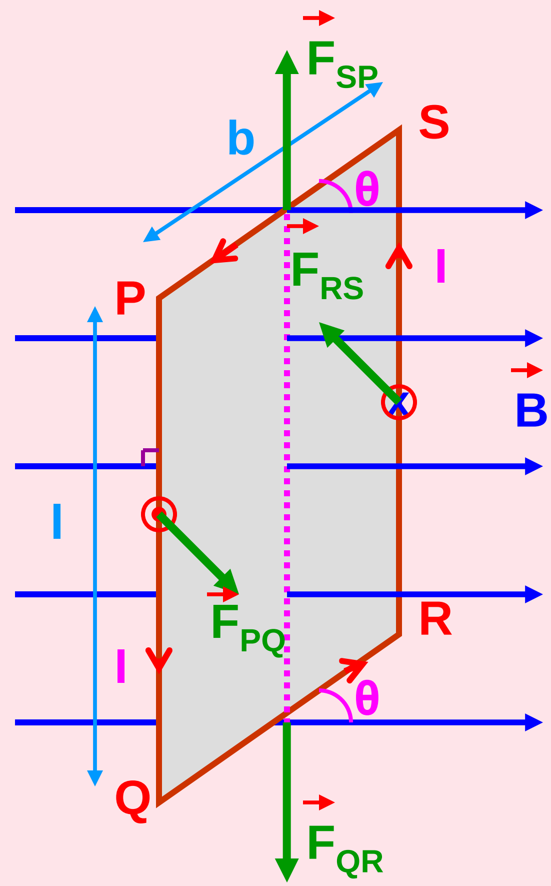
Representation of Field due to Parallel Currents:





Torque experienced by a Current Loop (Rectangular) in a uniform Magnetic Field:

Let θ be the angle between the plane of the loop and the direction of the magnetic field. The axis of the coil is perpendicular to the magnetic field.



$$\vec{F}_{SP} = I\,(\vec{b} \times \vec{B})$$

$$|\,F_{SP}\,| = I\,b\,B \sin\theta$$

$$\vec{F}_{QR} = I\,(\vec{b} \times \vec{B})$$

$$|\,F_{QR}\,| = I\,b\,B \sin\theta$$

Forces $\vec{F}_{SP}$ and $\vec{F}_{QR}$ are equal in magnitude but opposite in direction and they cancel out each other. Moreover they act along the same line of action (axis) and hence do not produce torque.

$$\vec{F}_{PQ} = I\,(\vec{l} \times \vec{B})$$

$$|\,F_{PQ}\,| = I\,l\,B \sin 90° = I\,l\,B$$

$$\vec{F}_{RS} = I\,(\vec{l} \times \vec{B})$$

$$|\,F_{Rs}\,| = I\,l\,B \sin 90° = I\,l\,B$$

Forces $\vec{F}_{PQ}$ and $\vec{F}_{RS}$ being equal in magnitude but opposite in direction cancel out each other and do not produce any translational motion. But they act along different lines of action and hence produce torque about the axis of the coil.

Torque experienced by the coil is

$\tau = F_{PQ} \times PN$ (in magnitude)

$\tau = I\, l\, B\, (b \cos\theta)$

$\tau = I\, lb\, B \cos\theta$

$\tau = I\, A\, B \cos\theta$ (A = lb)

$\tau = N\, I\, A\, B \cos\theta$ (where N is the no. of turns)

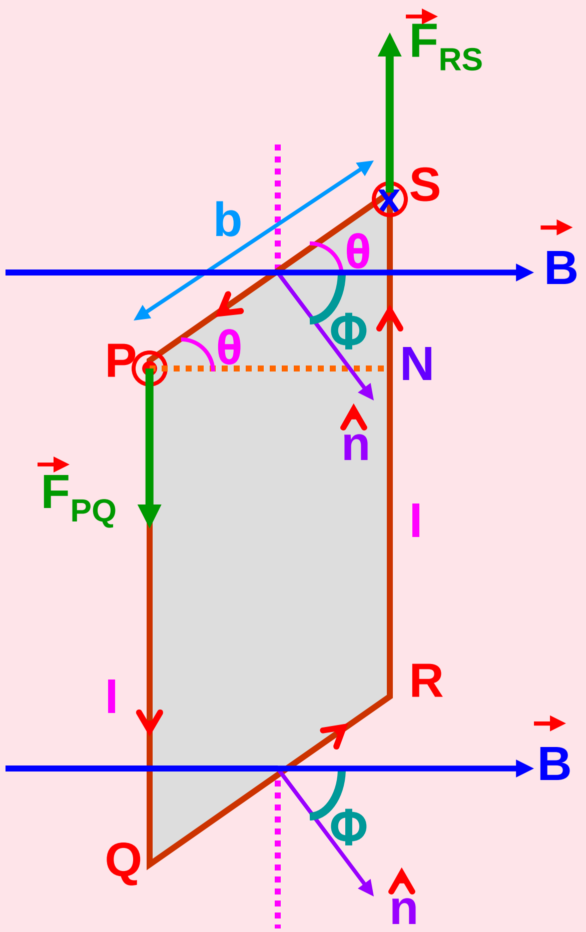
If Φ is the angle between the normal to the coil and the direction of the magnetic field, then

$\Phi + \theta = 90°$ i.e. $\theta = 90° - \Phi$

So,

$\tau = I\, A\, B \cos(90° - \Phi)$

$\tau = N\, I\, A\, B \sin\Phi$



NOTE:

One must be very careful in using the formula in terms of cos or sin since it depends on the angle taken whether with the plane of the coil or the normal of the coil.

Torque in Vector form:

$\tau = N\ I\ A\ B \sin \Phi$

$\vec{\tau} = (N\ I\ A\ B \sin \Phi)\ \hat{n}$ (where $\hat{n}$ is unit vector normal to the plane of the loop)

$\boxed{\vec{\tau} = N\ I\ (\vec{A} \times \vec{B})}$ or $\boxed{\vec{\tau} = N\ (\vec{M} \times \vec{B})}$

(since $\vec{M} = I\ \vec{A}$ is the Magnetic Dipole Moment)

Note:

1) The coil will rotate in the anticlockwise direction (from the top view, according to the figure) about the axis of the coil shown by the dotted line.
2) The torque acts in the upward direction along the dotted line (according to Maxwell's Screw Rule).
3) If $\Phi = 0°$, then $\tau = 0$.
4) If $\Phi = 90°$, then τ is maximum. i.e. $\tau_{max} = N\ I\ A\ B$
5) Units: B in Tesla, I in Ampere, A in m^2 and τ in Nm.
6) The above formulae for torque can be used for any loop irrespective of its shape.

Moving Coil or Suspended Coil or D' Arsonval Type Galvanometer:

Torque experienced by the coil is

$\tau = N\,I\,A\,B\sin\Phi$

Restoring torque in the coil is

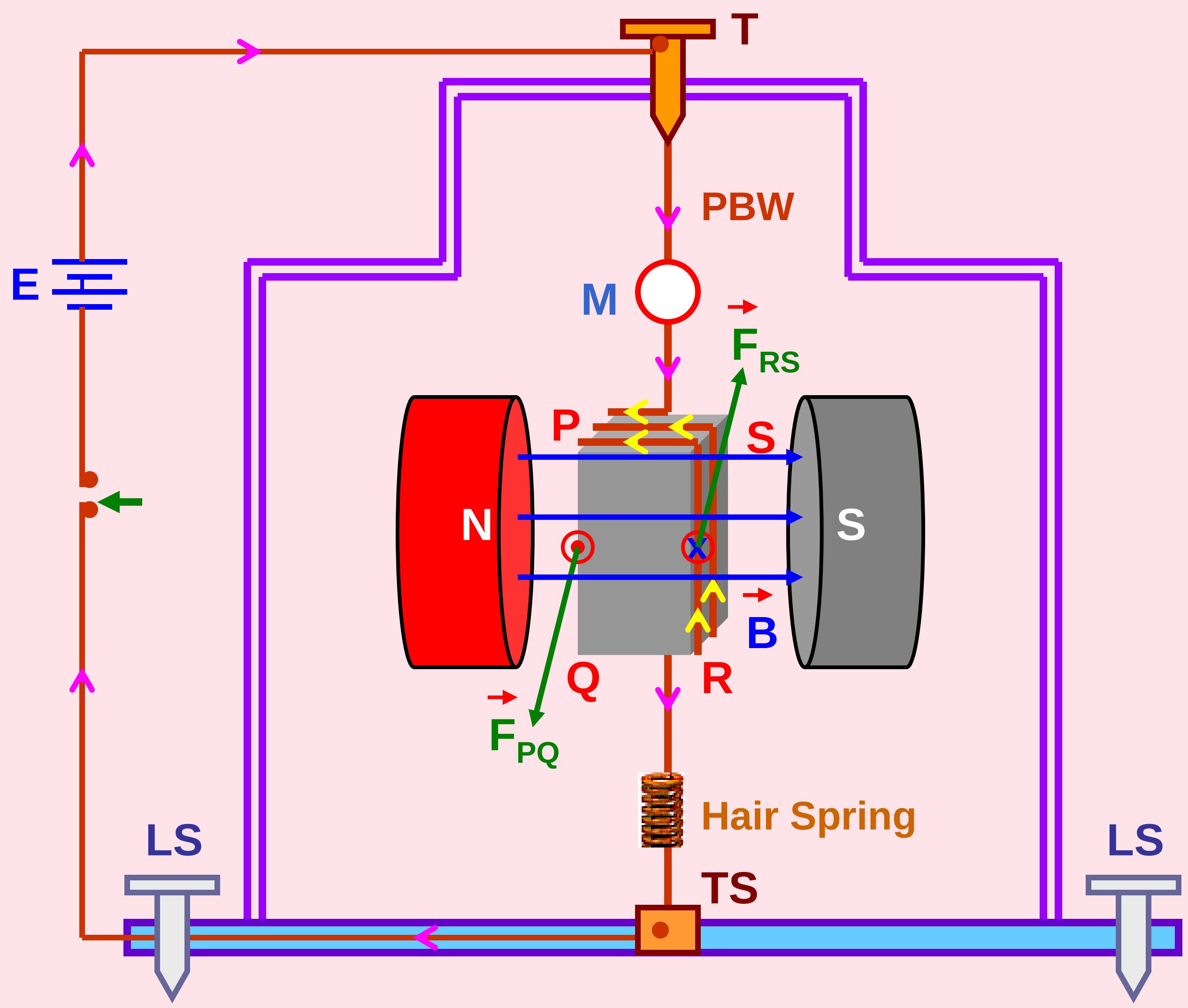
$\tau = k\,\alpha$ (where k is restoring torque per unit angular twist, α is the angular twist in the wire)

At equilibrium,

$N\,I\,A\,B\sin\Phi = k\,\alpha$

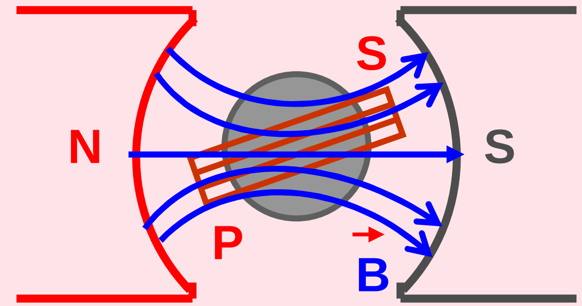
$\therefore\quad I = \dfrac{k}{N\,A\,B\sin\Phi}\,\alpha$

The factor sin Φ can be eliminated by choosing Radial Magnetic Field.



T – Torsion Head, TS – Terminal screw, M – Mirror, N,S – Poles pieces of a magnet, LS – Levelling Screws, PQRS – Rectangular coil, PBW – Phosphor Bronze Wire

Radial Magnetic Field:

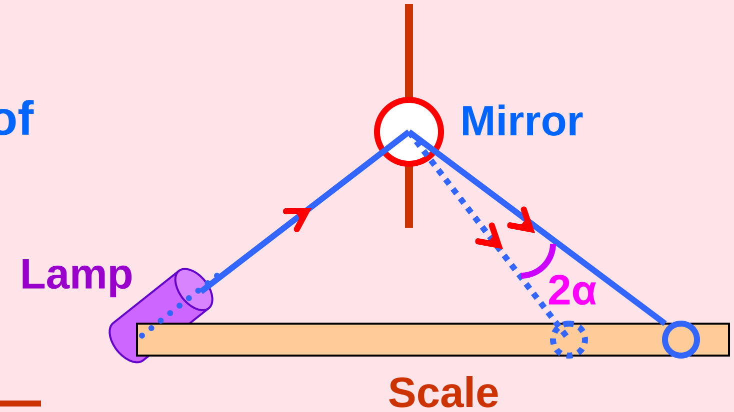


The (top view PS of) plane of the coil PQRS lies along the magnetic lines of force in whichever position the coil comes to rest in equilibrium.

So, the angle between the plane of the coil and the magnetic field is 0°.

or the angle between the normal to the plane of the coil and the magnetic field is 90°.

i.e. $\sin \Phi = \sin 90° = 1$



$$\therefore \boxed{I = \frac{k}{N\,A\,B}\,\alpha} \quad \text{or} \quad \boxed{I = G\,\alpha} \quad \text{where } G = \frac{k}{N\,A\,B}$$

is called Galvanometer constant

Current Sensitivity of Galvanometer:

It is the defection of galvanometer per unit current.

$$\boxed{\frac{\alpha}{I} = \frac{N\,A\,B}{k}}$$

Voltage Sensitivity of Galvanometer:

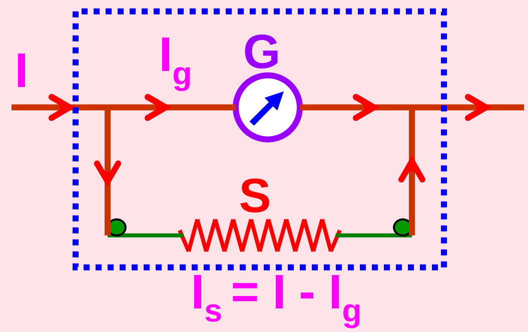
It is the defection of galvanometer per unit voltage.

$$\boxed{\frac{\alpha}{V} = \frac{N\,A\,B}{kR}}$$

Conversion of Galvanometer to Ammeter:

Galvanometer can be converted into ammeter by shunting it with a very small resistance.

Potential difference across the galvanometer and shunt resistance are equal.

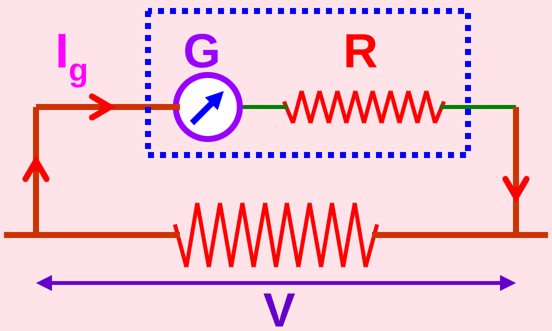


$$\therefore \quad (I - I_g)\, S = I_g\, G \qquad \text{or} \qquad S = \frac{I_g\, G}{I - I_g}$$

Conversion of Galvanometer to Voltmeter:

Galvanometer can be converted into voltmeter by connecting it with a very high resistance.

Potential difference across the given load resistance is the sum of p.d across galvanometer and p.d. across the high resistance.



$$\therefore \quad V = I_g\,(G + R) \qquad \text{or} \qquad R = \frac{V}{I_g} - G$$

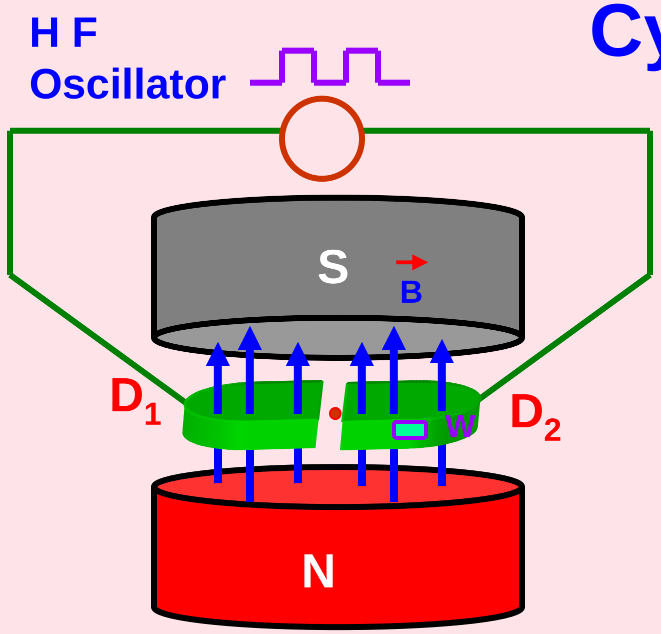
Difference between Ammeter and Voltmeter:

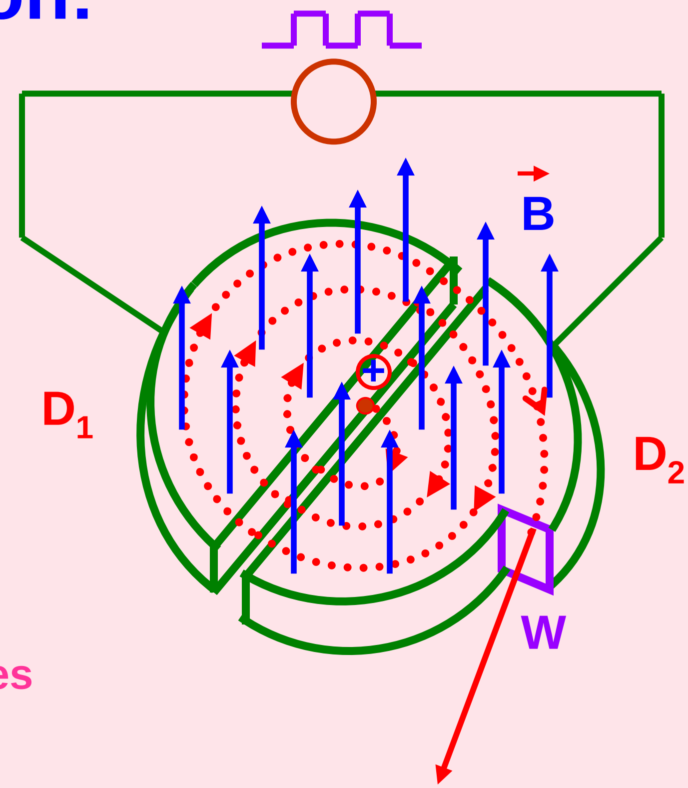
S.No.	Ammeter	Voltmeter
1	It is a low resistance instrument.	It is a high resistance instrument.
2	Resistance is GS / (G + S)	Resistance is G + R
3	Shunt Resistance is $(GI_g) / (I - I_g)$ and is very small.	Series Resistance is $(V / I_g) - G$ and is very high.
4	It is always connected in series.	It is always connected in parallel.
5	Resistance of an ideal ammeter is zero.	Resistance of an ideal voltmeter is infinity.
6	Its resistance is less than that of the galvanometer.	Its resistance is greater than that of the voltmeter.
7	It is not possible to decrease the range of the given ammeter.	It is possible to decrease the range of the given voltmeter.

MAGNETIC EFFECT OF CURRENT - III

1. Cyclotron
2. Ampere's Circuital Law
3. Magnetic Field due to a Straight Solenoid
4. Magnetic Field due to a Toroidal Solenoid

Cyclotron:





D_1, D_2 – Dees   N, S – Magnetic Pole Pieces

W – Window   B - Magnetic Field

Working: Imagining D_1 is positive and D_2 is negative, the + vely charged particle kept at the centre and in the gap between the dees get accelerated towards D_2. Due to perpendicular magnetic field and according to Fleming's Left Hand Rule the charge gets deflected and describes semi-circular path.

When it is about to leave D_2, D_2 becomes + ve and D_1 becomes – ve. Therefore the particle is again accelerated into D_1 where it continues to describe the semi-circular path. The process continues till the charge traverses through the whole space in the dees and finally it comes out with very high speed through the window.

Theory:

The magnetic force experienced by the charge provides centripetal force required to describe circular path.

$\therefore \quad mv^2 / r = qvB \sin 90°$

$$v = \frac{B\,q\,r}{m}$$

(where m – mass of the charged particle, q – charge, v – velocity on the path of radius – r, B is magnetic field and 90° is the angle b/n v and B)

If t is the time taken by the charge to describe the semi-circular path inside the dee, then

$$t = \frac{\pi\ r}{v} \quad \text{or} \quad t = \frac{\pi\, m}{B\,q}$$

Time taken inside the dee depends only on the magnetic field and m/q ratio and not on the speed of the charge or the radius of the path.

If T is the time period of the high frequency oscillator, then for resonance,

$$T = 2\,t \quad \text{or} \quad T = \frac{2\pi m}{B\,q}$$

If f is the frequency of the high frequency oscillator (Cyclotron Frequency), then

$$\boxed{f = \frac{B\,q}{2\pi m}}$$

Maximum Energy of the Particle:

Kinetic Energy of the charged particle is

$$K.E. = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{B\,q\,r}{m}\right)^2 = \frac{1}{2} \frac{B^2 q^2 r^2}{m}$$

Maximum Kinetic Energy of the charged particle is when r = R (radius of the D's).

$$\boxed{K.E._{max} = \frac{1}{2} \frac{B^2 q^2 R^2}{m}}$$

The expressions for Time period and Cyclotron frequency only when m remains constant. (Other quantities are already constant.)

But m varies with v according to Einstein's Relativistic Principle as per

$$m = \frac{m_0}{[1 - (v^2 / c^2)]^{1/2}}$$

If frequency is varied in synchronisation with the variation of mass of the charged particle (by maintaining B as constant) to have resonance, then the cyclotron is called synchro – cyclotron.

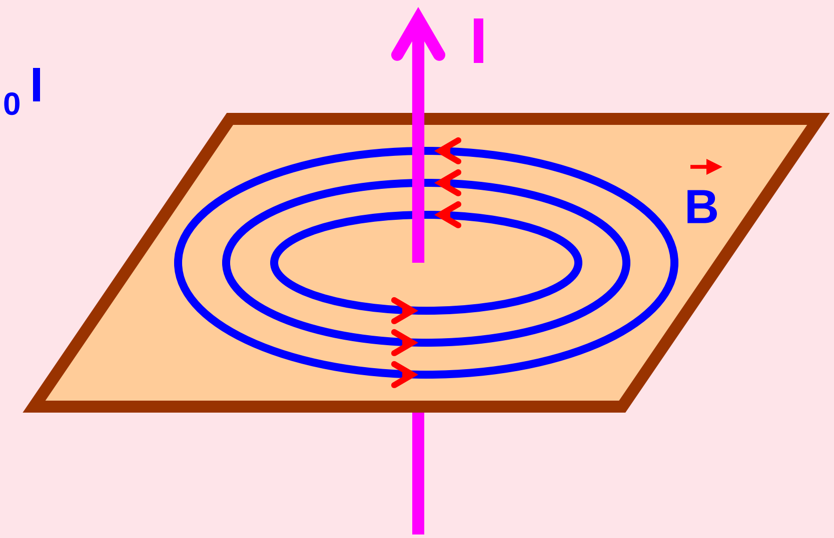
If magnetic field is varied in synchronisation with the variation of mass of the charged particle (by maintaining f as constant) to have resonance, then the cyclotron is called isochronous – cyclotron.

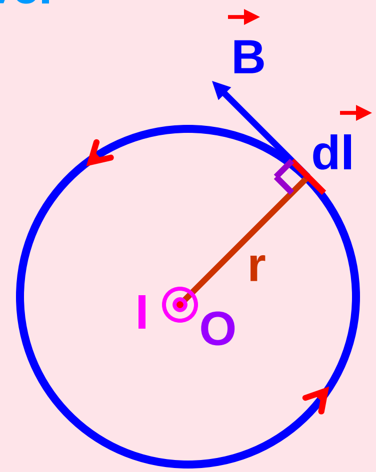
NOTE: Cyclotron can not be used for accelerating neutral particles. Electrons can not be accelerated because they gain speed very quickly due to their lighter mass and go out of phase with alternating e.m.f. and get lost within the dees.

Ampere's Circuital Law:

The line integral $\oint \vec{B} \cdot \vec{dl}$ for a closed curve is equal to μ_0 times the net current I threading through the area bounded by the curve.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$





Current is emerging out and the magnetic field is anticlockwise.

Proof:

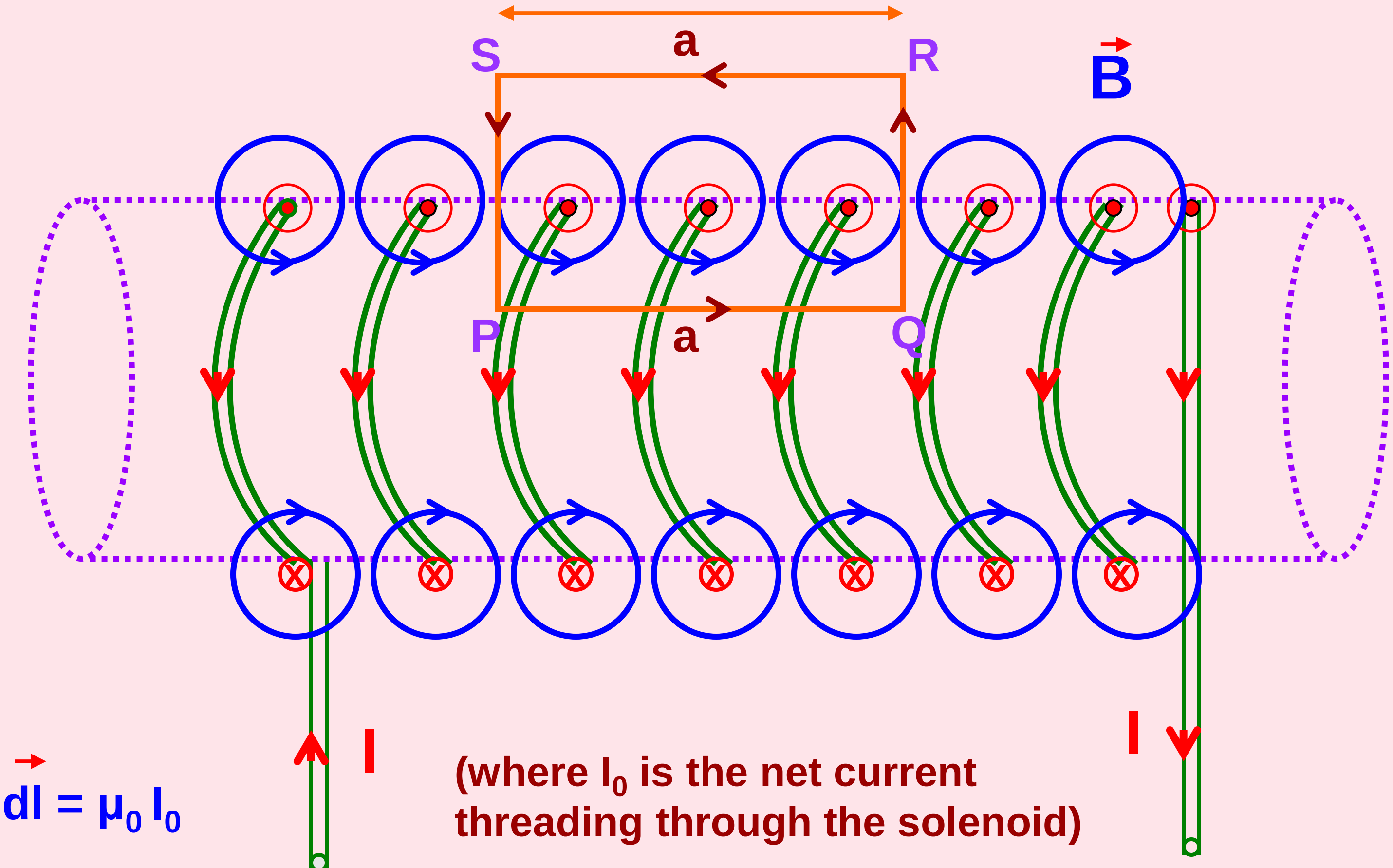
$$\oint \vec{B} \cdot \vec{dl} = \oint B \, . \, dl \cos 0°$$

$$= \oint B \, . \, dl = B \oint dl$$

$$= B\,(2\pi\, r) = (\mu_0 I / 2\pi\, r) \times 2\pi\, r$$

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I}$$

Magnetic Field at the centre of a Straight Solenoid:



$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_0$

(where I_0 is the net current threading through the solenoid)

$$\oint \vec{B} \cdot \vec{dl} = \oint_{PQ} \vec{B} \cdot \vec{dl} + \oint_{QR} \vec{B} \cdot \vec{dl} + \oint_{RS} \vec{B} \cdot \vec{dl} + \oint_{SP} \vec{B} \cdot \vec{dl}$$

$$= \oint B \cdot dl \cos 0° + \oint B \cdot dl \cos 90° + \oint 0 \cdot dl \cos 0° + \oint B \cdot dl \cos 90°$$

$$= B \oint dl = B.a \quad \text{and} \quad \mu_0 I_0 = \mu_0 n a I \quad \therefore \quad \boxed{B = \mu_0 n I}$$

(where n is no. of turns per unit length, a is the length of the path and I is the current passing through the lead of the solenoid)

Magnetic Field due to Toroidal Solenoid (Toroid):

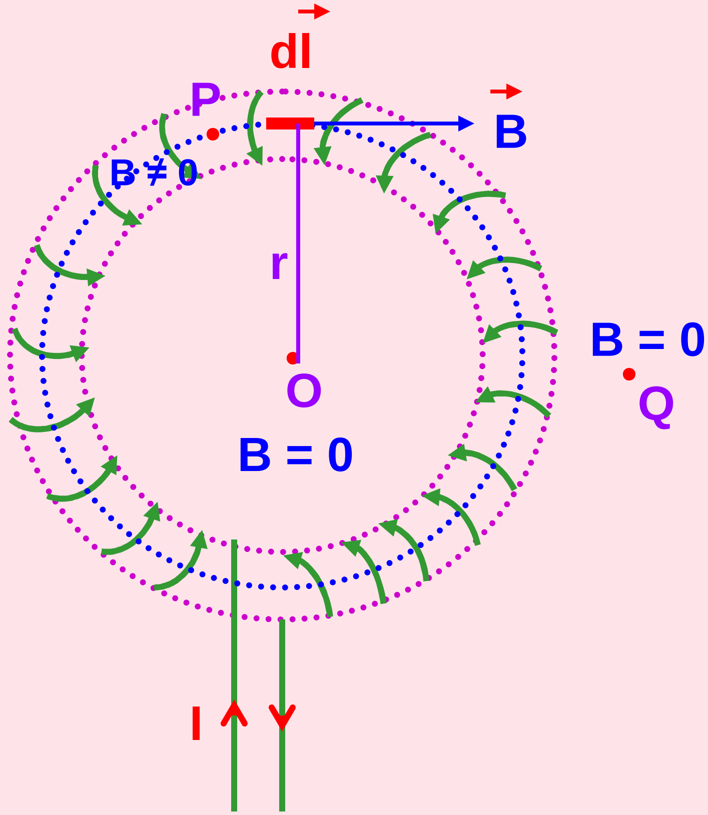
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_0$$

$$\oint \vec{B} \cdot \vec{dl} = \oint B \cdot dl \cos 0°$$

$$= B \oint dl = B\,(2\pi\ r)$$

And $\mu_0 I_0 = \mu_0 n\,(2\pi\ r)\, I$

$$\therefore \quad \boxed{B = \mu_0 n\ I}$$



NOTE:

The magnetic field exists only in the tubular area bound by the coil and it does not exist in the area inside and outside the toroid.

i.e. B is zero at O and Q and non-zero at P.

MAGNETISM

1. Bar Magnet and its properties
2. Current Loop as a Magnetic Dipole and Dipole Moment
3. Current Solenoid equivalent to Bar Magnet
4. Bar Magnet and it Dipole Moment
5. Coulomb’s Law in Magnetism
6. Important Terms in Magnetism
7. Magnetic Field due to a Magnetic Dipole
8. Torque and Work Done on a Magnetic Dipole
9. Terrestrial Magnetism
10. Elements of Earth’s Magnetic Field
11. Tangent Law
12. Properties of Dia-, Para- and Ferro-magnetic substances
13. Curie’s Law in Magnetism
14. Hysteresis in Magnetism

Magnetism:

- Phenomenon of attracting magnetic substances like iron, nickel, cobalt, etc.

- **A body possessing the property of magnetism is called a magnet.**
- **A magnetic pole is a point near the end of the magnet where magnetism is concentrated.**
- **Earth is a natural magnet.**
- **The region around a magnet in which it exerts forces on other magnets and on objects made of iron is a magnetic field.**

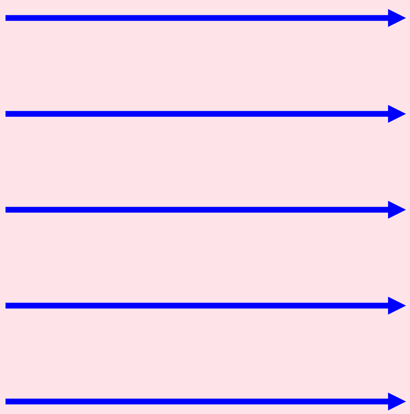
Properties of a bar magnet:

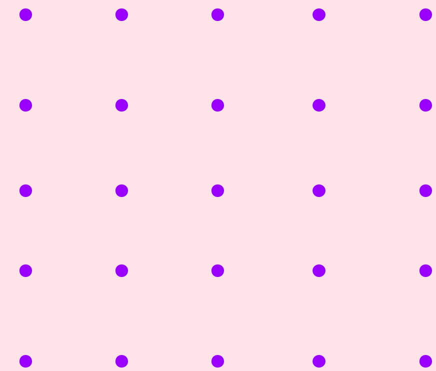
1. **A freely suspended magnet aligns itself along North – South direction.**
2. **Unlike poles attract and like poles repel each other.**
3. **Magnetic poles always exist in pairs. i.e. Poles can not be separated.**
4. **A magnet can induce magnetism in other magnetic substances.**
5. **It attracts magnetic substances.**

Repulsion is the surest test of magnetisation: **A magnet attracts iron rod as well as opposite pole of other magnet. Therefore it is not a sure test of magnetisation.**

But, if a rod is repelled with strong force by a magnet, then the rod is surely magnetised.

Representation of Uniform Magnetic Field:



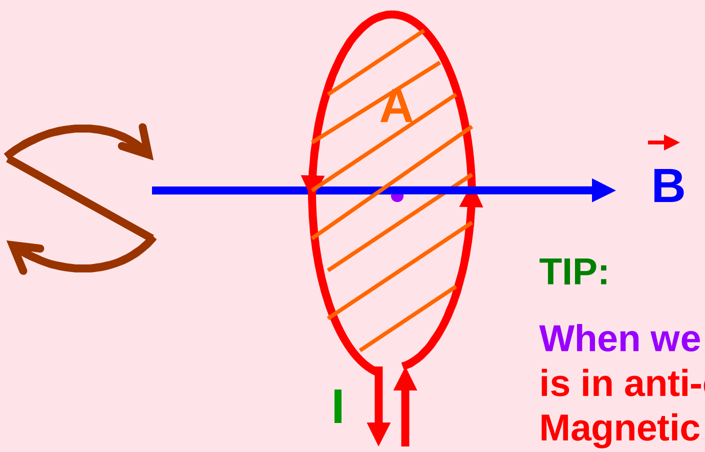


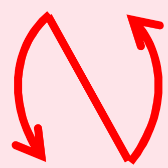
Uniform field on the plane of the diagram

Uniform field perpendicular & into the plane of the diagram

Uniform field perpendicular & emerging out of the plane of the diagram

Current Loop as a Magnetic Dipole & Dipole Moment:





Magnetic Dipole Moment is

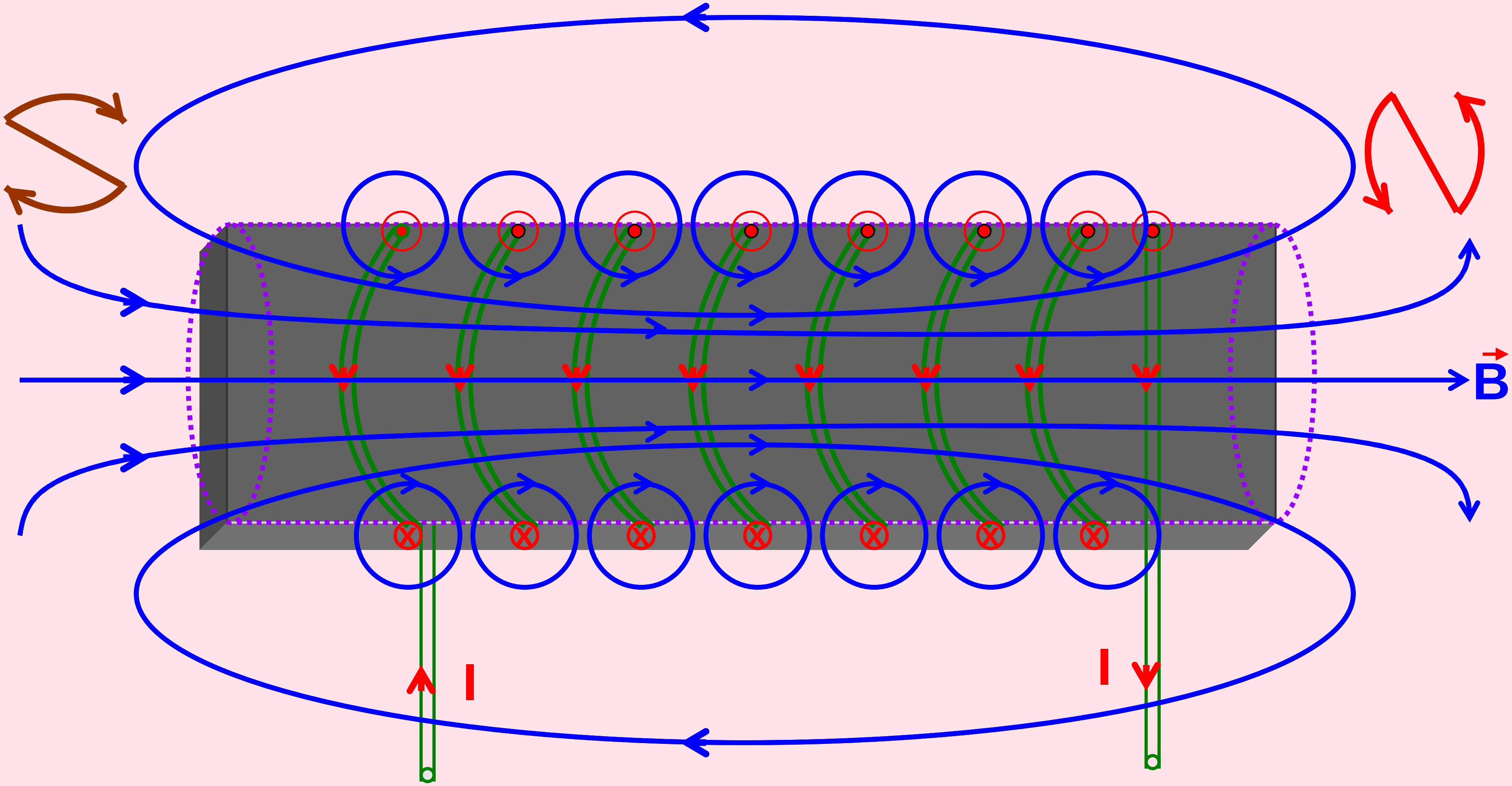
$$\boxed{\vec{M} = I\,A\,\hat{n}}$$

SI unit is A m^2.

TIP:

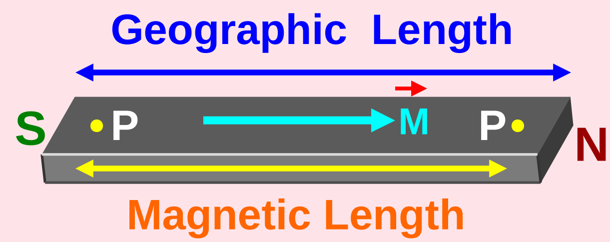
When we look at any one side of the loop carrying current, if the current is in anti-clockwise direction then that side of the loop behaves like Magnetic North Pole and if the current is in clockwise direction then that side of the loop behaves like Magnetic South Pole.

Current Solenoid as a Magnetic Dipole or Bar Magnet:



TIP: **Play previous and next to understand the similarity of field lines.**

Bar Magnet:



1. The line joining the poles of the magnet is called magnetic axis.
2. The distance between the poles of the magnet is called magnetic length of the magnet.
3. The distance between the ends of the magnet is called the geometrical length of the magnet.
4. The ratio of magnetic length and geometrical length is nearly 0.84.

Magnetic Dipole & Dipole Moment:

A pair of magnetic poles of equal and opposite strengths separated by a finite distance is called a magnetic dipole.

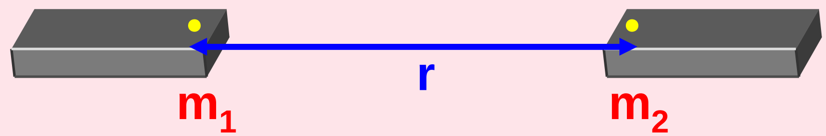
The magnitude of dipole moment is the product of the pole strength m and the separation 2l between the poles.

Magnetic Dipole Moment is $\boxed{\vec{M} = m.2l.\hat{l}}$ SI unit of pole strength is A.m

The direction of the dipole moment is from South pole to North Pole along the axis of the magnet.

Coulomb's Law in Magnetism:

The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.



$F \,\alpha\, m_1\, m_2$

$\alpha\, r^2$

$$F = \frac{k\, m_1\, m_2}{r^2} \quad \text{or} \quad \boxed{F = \frac{\mu_0\, m_1\, m_2}{4\pi \quad r^2}}$$

(where $k = \mu_0 / 4\pi$ is a constant and $\mu_0 = 4\pi \times 10^{-7}$ T m A-1)

In <u>vector form</u>

$$\vec{F} = \frac{\mu_0\, m_1\, m_2\, \hat{r}}{4\pi \quad r^2}$$

$$\boxed{\vec{F} = \frac{\mu_0\, m_1\, m_2\, \vec{r}}{4\pi \quad r^3}}$$

Magnetic Intensity or Magnetising force (H):

i) Magnetic Intensity at a point is the force experienced by a north pole of unit pole strength placed at that point due to pole strength of the given magnet. $H = B / \mu$

ii) It is also defined as the magnetomotive force per unit length.

iii) It can also be defined as the degree or extent to which a magnetic field can magnetise a substance.

iv) It can also be defined as the force experienced by a unit positive charge flowing with unit velocity in a direction normal to the magnetic field.

v) Its SI unit is ampere-turns per linear metre.

vi) Its cgs unit is oersted.

Magnetic Field Strength or Magnetic Field or Magnetic Induction or Magnetic Flux Density (B):

i) Magnetic Flux Density is the number of magnetic lines of force passing normally through a unit area of a substance. $B = \mu H$

ii) Its SI unit is weber-m^{-2} or Tesla (T).

iii) Its cgs unit is gauss. 1 gauss = 10^{-4} Tesla

Magnetic Flux (Φ):

i) It is defined as the number of magnetic lines of force passing normally through a surface.

ii) Its SI unit is weber.

Relation between B and H:

$B = \mu H$ (where μ is the permeability of the medium)

Magnetic Permeability (μ):

It is the degree or extent to which magnetic lines of force can pass enter a substance.

Its SI unit is $T\ m\ A^{-1}$ or $wb\ A^{-1}\ m^{-1}$ or $H\ m^{-1}$

Relative Magnetic Permeability (μ_r):

It is the ratio of magnetic flux density in a material to that in vacuum.

It can also be defined as the ratio of absolute permeability of the material to that in vacuum.

$\mu_r = B / B_0$ or $\mu_r = \mu / \mu_0$

Intensity of Magnetisation: (I):

i) It is the degree to which a substance is magnetised when placed in a magnetic field.

ii) It can also be defined as the magnetic dipole moment (M) acquired per unit volume of the substance (V).

iii) It can also be defined as the pole strength (m) per unit cross-sectional area (A) of the substance.

iv) $I = M / V$

v) $I = m(2l) / A(2l) \quad = m / A$

vi) SI unit of Intensity of Magnetisation is $A\ m^{-1}$.

Magnetic Susceptibility (c_m):

i) It is the property of the substance which shows how easily a substance can be magnetised.

ii) It can also be defined as the ratio of intensity of magnetisation (I) in a substance to the magnetic intensity (H) applied to the substance.

iii) $c_m = I / H$

Susceptibility has no unit.

Relation between Magnetic Permeability (μ_r) & Susceptibility (c_m):

$$\mu_r = 1 + c_m$$

Magnetic Field due to a Magnetic Dipole (Bar Magnet):

i) At a point on the axial line of the magnet:

$$\boxed{B_P = \frac{\mu_0 \, 2 M x}{4\pi \, (x^2 - l^2)^2}}$$

If $l \ll x$, then

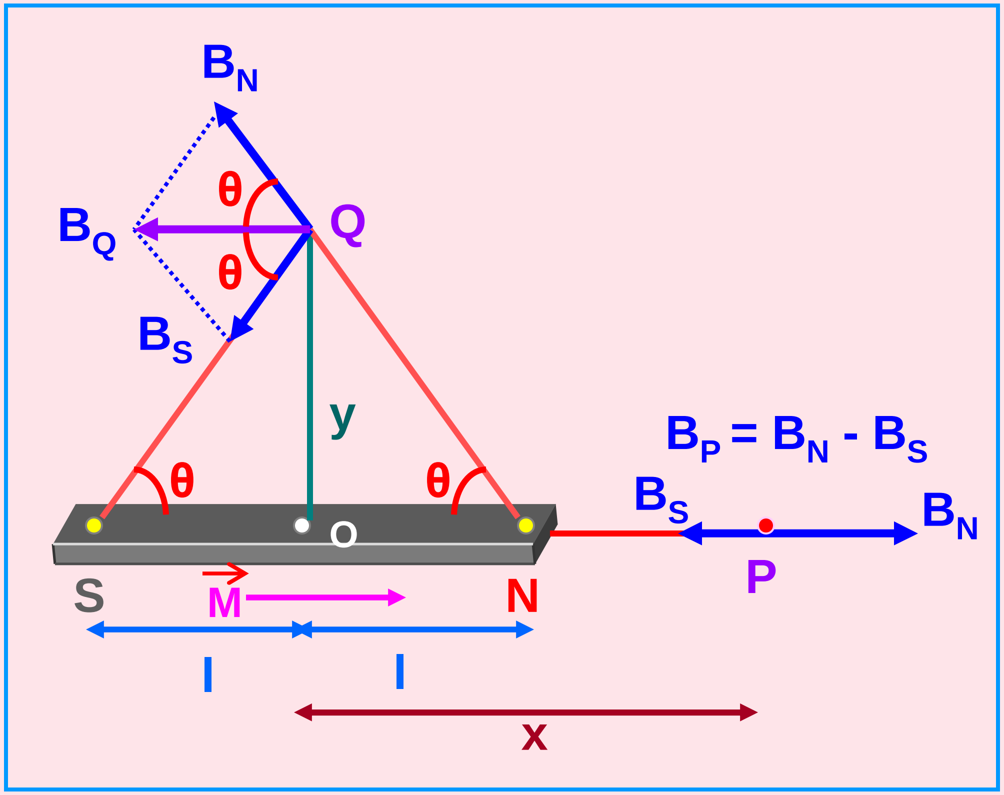
$$\boxed{B_P \approx \frac{\mu_0 \, 2 M}{4\pi \; x^3}}$$

ii) At a point on the equatorial line of the magnet:

$$\boxed{B_Q = \frac{\mu_0 \; M}{4\pi \, (y^2 + l^2)^{3/2}}}$$

If $l \ll y$, then

$$\boxed{B_P \approx \frac{\mu_0 \; M}{4\pi \; y^3}}$$

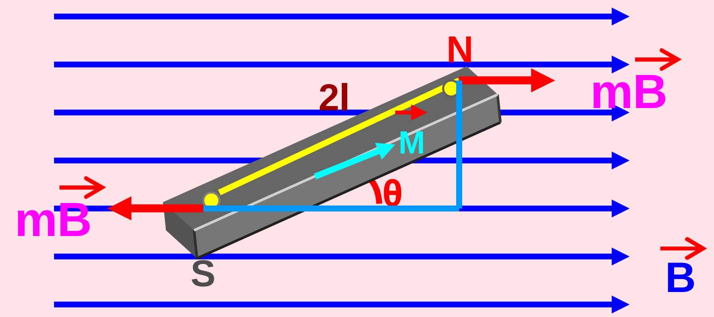


Magnetic Field at a point on the axial line acts along the dipole moment vector.

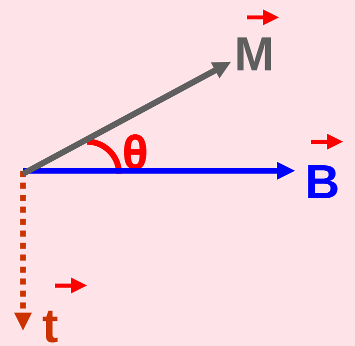
Magnetic Field at a point on the equatorial line acts opposite to the dipole moment vector.

Torque on a Magnetic Dipole (Bar Magnet) in Uniform Magnetic Field:

The forces of magnitude mB act opposite to each other and hence net force acting on the bar magnet due to external uniform magnetic field is zero. So, there is no translational motion of the magnet.



However the forces are along different lines of action and constitute a couple. Hence the magnet will rotate and experience torque.



Torque = Magnetic Force x ⊥ distance

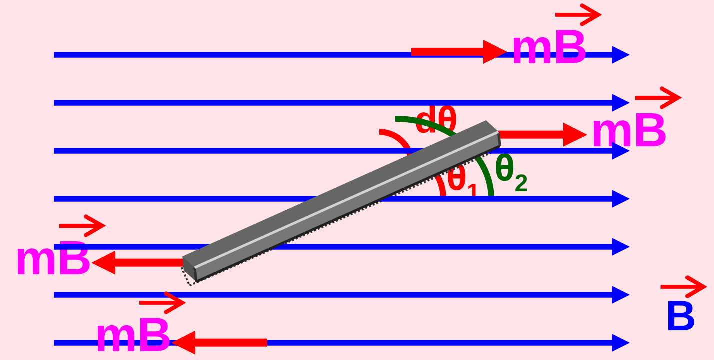
$$t = mB\,(2l \sin \theta)$$

$$= M B \sin \theta$$

$$\boxed{\vec{t} = \vec{M} \times \vec{B}}$$

Direction of Torque is perpendicular and into the plane containing $\vec{M}$ and $\vec{B}$.

Work done on a Magnetic Dipole (Bar Magnet) in Uniform Magnetic Field:



$dW = \tau d\theta$

$= M B \sin\theta \, d\theta$

$$W = \int_{\theta_1}^{\theta_2} M B \sin\theta \, d\theta$$

$$W = M B (\cos\theta_1 - \cos\theta_2)$$

If Potential Energy is arbitrarily taken zero when the dipole is at 90°, then P.E in rotating the dipole and inclining it at an angle θ is

Potential Energy = $- M B \cos\theta$

Note:

Potential Energy can be taken zero arbitrarily at any position of the dipole.

Terrestrial Magnetism:

i) **Geographic Axis** is a straight line passing through the geographical poles of the earth. It is the axis of rotation of the earth. It is also known as polar axis.

ii) **Geographic Meridian** at any place is a vertical plane passing through the geographic north and south poles of the earth.

iii) **Geographic Equator** is a great circle on the surface of the earth, in a plane perpendicular to the geographic axis. All the points on the geographic equator are at equal distances from the geographic poles.

iv) **Magnetic Axis** is a straight line passing through the magnetic poles of the earth. It is inclined to Geographic Axis nearly at an angle of 17°.

v) **Magnetic Meridian** at any place is a vertical plane passing through the magnetic north and south poles of the earth.

vi) **Magnetic Equator** is a great circle on the surface of the earth, in a plane perpendicular to the magnetic axis. All the points on the magnetic equator are at equal distances from the magnetic poles.

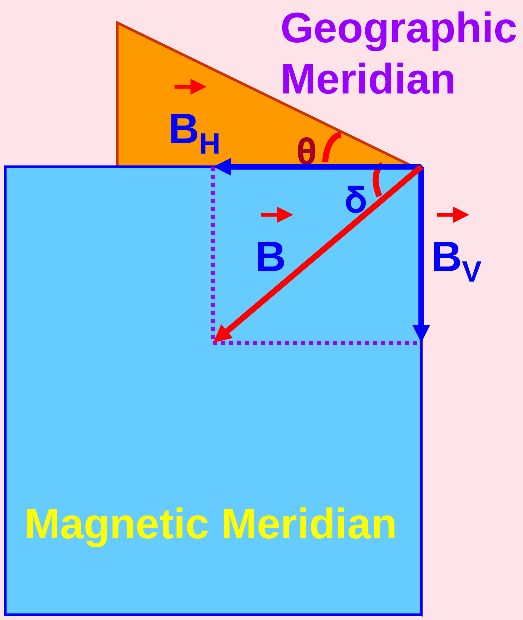
Declination (θ):

The angle between the magnetic meridian and the geographic meridian at a place is Declination at that place.

It varies from place to place.

Lines shown on the map through the places that have the same declination are called isogonic line.

Line drawn through places that have zero declination is called an agonic line.



Dip or Inclination (δ):

The angle between the horizontal component of earth's magnetic field and the earth's resultant magnetic field at a place is Dip or Inclination at that place.

It is zero at the equator and 90° at the poles.

Lines drawn up on a map through places that have the same dip are called isoclinic lines.

The line drawn through places that have zero dip is known as an aclinic line. It is the magnetic equator.

Horizontal Component of Earth's Magnetic Field (B_H):

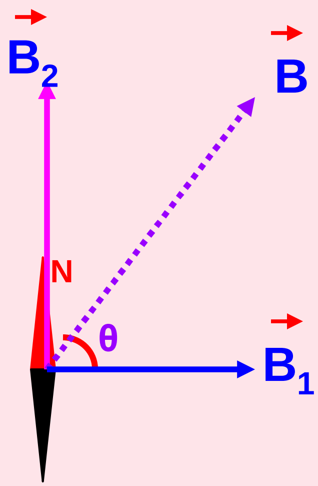
The total intensity of the earth's magnetic field does not lie in any horizontal plane. Instead, it lies along the direction at an angle of dip (δ) to the horizontal. The component of the earth's magnetic field along the horizontal at an angle δ is called Horizontal Component of Earth's Magnetic Field.

$$B_H = B \cos \delta$$

Similarly Vertical Component is $B_V = B \sin \delta$

such that $B = \sqrt{B_H^2 + B_V^2}$

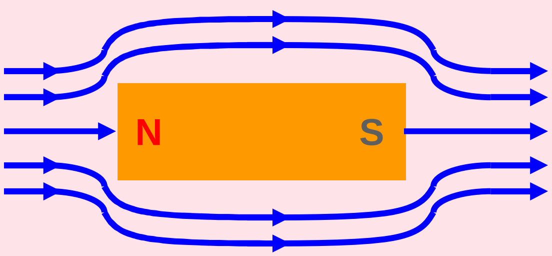
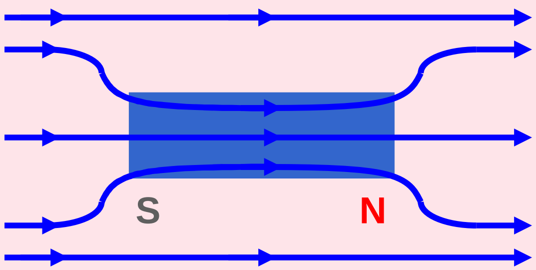
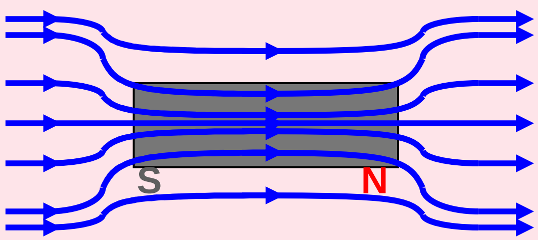
Tangent Law:

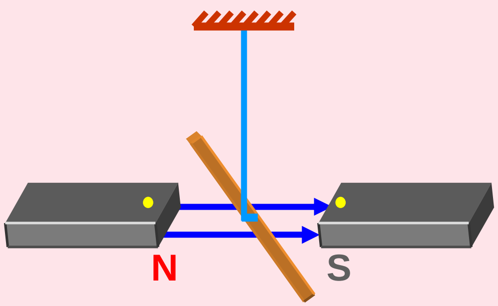
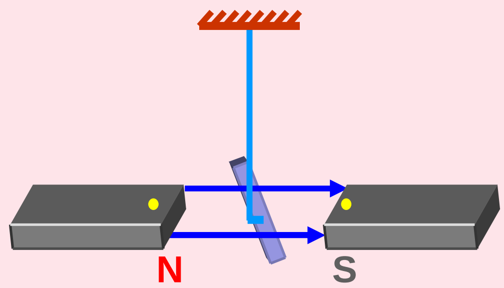
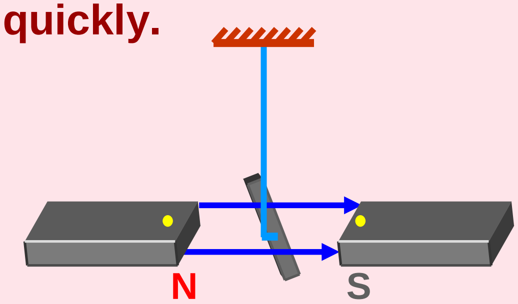


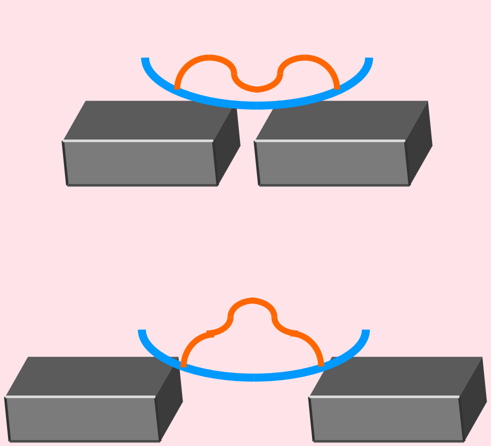
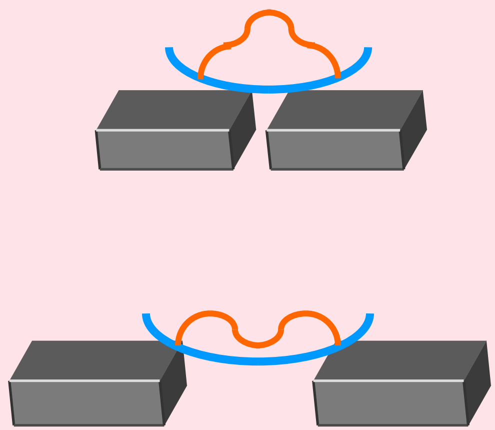
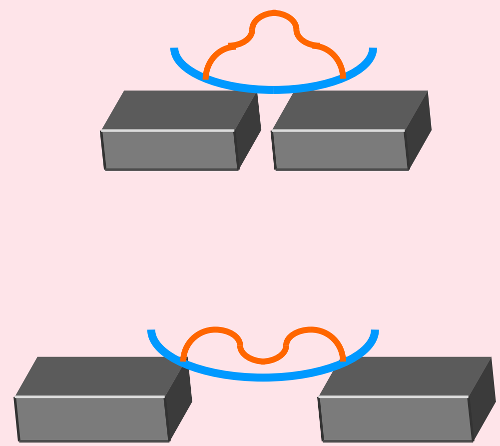
If a magnetic needle is suspended in a region where two uniform magnetic fields are perpendicular to each other, the needle will align itself along the direction of the resultant field of the two fields at an angle θ such that the tangent of the angle is the ratio of the two fields.

$$\boxed{\tan \theta = B_2 / B_1}$$

Comparison of Dia, Para and Ferro Magnetic materials:

DIA	PARA	FERRO
1. Diamagnetic substances are those substances which are feebly repelled by a magnet. Eg. Antimony, Bismuth, Copper, Gold, Silver, Quartz, Mercury, Alcohol, water, Hydrogen, Air, Argon, etc.	Paramagnetic substances are those substances which are feebly attracted by a magnet. Eg. Aluminium, Chromium, Alkali and Alkaline earth metals, Platinum, Oxygen, etc.	Ferromagnetic substances are those substances which are strongly attracted by a magnet. Eg. Iron, Cobalt, Nickel, Gadolinium, Dysprosium, etc.
2. When placed in magnetic field, the lines of force tend to avoid the substance. 	The lines of force prefer to pass through the substance rather than air. 	The lines of force tend to crowd into the specimen. 

2. When placed in non-uniform magnetic field, it moves from stronger to weaker field (feeble repulsion).	When placed in non-uniform magnetic field, it moves from weaker to stronger field (feeble attraction).	When placed in non-uniform magnetic field, it moves from weaker to stronger field (strong attraction).
3. When a diamagnetic rod is freely suspended in a uniform magnetic field, it aligns itself in a direction perpendicular to the field. 	When a paramagnetic rod is freely suspended in a uniform magnetic field, it aligns itself in a direction parallel to the field. 	When a paramagnetic rod is freely suspended in a uniform magnetic field, it aligns itself in a direction parallel to the field very quickly. 

4. If diamagnetic liquid taken in a watch glass is placed in uniform magnetic field, it collects away from the centre when the magnetic poles are closer and collects at the centre when the magnetic poles are farther.	If paramagnetic liquid taken in a watch glass is placed in uniform magnetic field, it collects at the centre when the magnetic poles are closer and collects away from the centre when the magnetic poles are farther.	If ferromagnetic liquid taken in a watch glass is placed in uniform magnetic field, it collects at the centre when the magnetic poles are closer and collects away from the centre when the magnetic poles are farther.
		

5. When a diamagnetic substance is placed in a magnetic field, it is weakly magnetised in the direction opposite to the inducing field.	When a paramagnetic substance is placed in a magnetic field, it is weakly magnetised in the direction of the inducing field.	When a ferromagnetic substance is placed in a magnetic field, it is strongly magnetised in the direction of the inducing field.
6. Induced Dipole Moment (M) is a small – ve value.	Induced Dipole Moment (M) is a small + ve value.	Induced Dipole Moment (M) is a large + ve value.
7. Intensity of Magnetisation (I) has a small – ve value.	Intensity of Magnetisation (I) has a small + ve value.	Intensity of Magnetisation (I) has a large + ve value.
8. Magnetic permeability μ is always less than unity.	Magnetic permeability μ is more than unity.	Magnetic permeability μ is large i.e. much more than unity.

9. Magnetic susceptibility c_m has a small – ve value.	Magnetic susceptibility c_m has a small + ve value.	Magnetic susceptibility c_m has a large + ve value.
10. They do not obey Curie's Law. i.e. their properties do not change with temperature.	They obey Curie's Law. They lose their magnetic properties with rise in temperature.	They obey Curie's Law. At a certain temperature called Curie Point, they lose ferromagnetic properties and behave like paramagnetic substances.

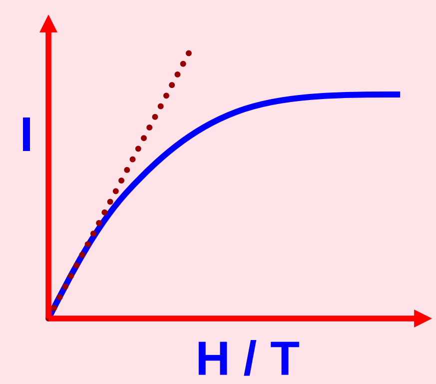
Curie's Law:

Magnetic susceptibility of a material varies inversely with the absolute temperature.

$I \alpha H / T$ or $I / H \alpha 1 / T$

$c_m \alpha 1 / T$

$c_m = C / T$ (where C is Curie constant)



Curie temperature for iron is 1000 K, for cobalt 1400 K and for nickel 600 K.

Hysteresis Loop or Magnetisation Curve:

Intensity of Magnetisation (I) increases with increase in Magnetising Force (H) initially through OA and reaches saturation at A.

When H is decreased, I decreases but it does not come to zero at H = 0.

The residual magnetism (I) set up in the material represented by OB is called Retentivity.

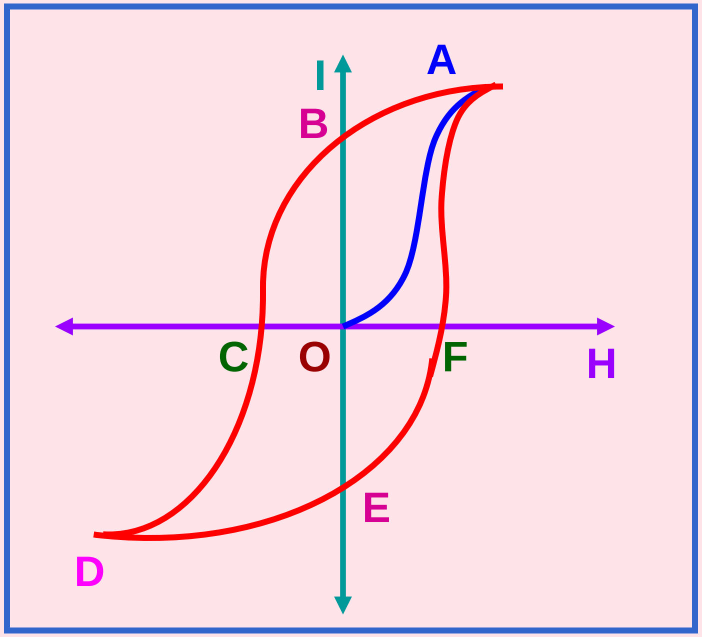
To bring I to zero (to demagnetise completely), opposite (negative) magnetising force is applied. This magetising force represented by OC is called coercivity.

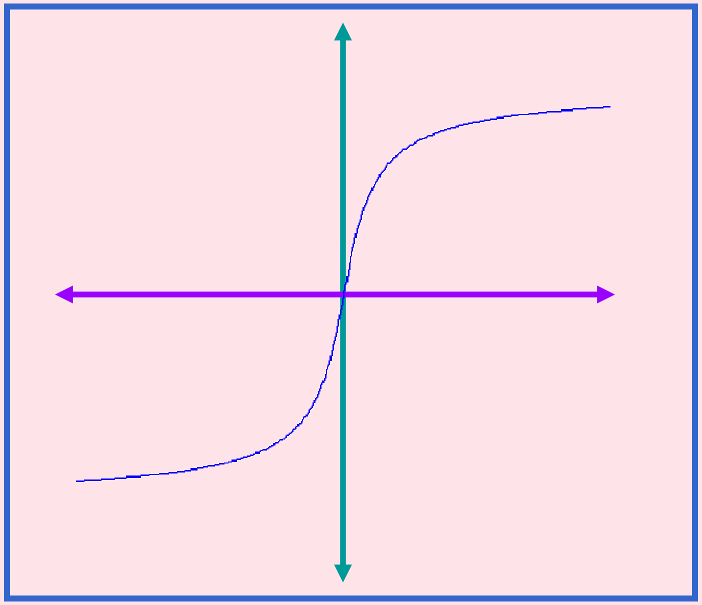
After reaching the saturation level D, when the magnetising force is reversed, the curve closes to the point A completing a cycle.

The loop ABCDEFA is called Hysteresis Loop.

The area of the loop gives the loss of energy due to the cycle of magnetisation and demagnetisation and is dissipated in the form of heat.

The material (like iron) having thin loop is used for making temporary magnets and that with thick loop (like steel) is used for permanent magnets.





"ORGANIC CHEMISTRY"

ALCOHOLS, PHENOLS AND ETHERS

An alcohol contains one or more hydroxyl (OH) group(s) directly attached to carbon atom(s), of an aliphatic system. e.g. C_2H_5OH (Ethanol)

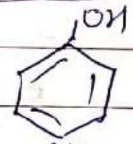
A phenol contains $-OH$ group(s) directly attached to carbon atom(s) of an aromatic system (benzene ring). e.g. C_6H_5OH (Phenol)

Ethers are a class of compounds formed by the substitution of a hydrogen atom in a hydrocarbon by an alkoxy or aryloxy group (R-O/Ar-O). e.g. CH_3OCH_3 (dimethyl ether).

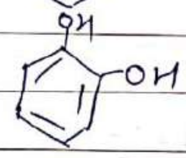
CLASSIFICATION

1. MONO, DI, TRI or POLYHYDRIC COMPOUNDS :-

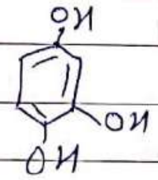
MONOHYDRIC C_2H_5OH



DIHYDRIC CH_2OH-CH_2OH



TRIHYDRIC $CH_2OH-CHOH-CH_2OH$



FURTHER CLASSIFICATION OF MONOHYDRIC ALCOHOLS

(i) Compounds containing $C_{sp^3}-OH$ bond :-

(a) Primary, secondary and tertiary alcohols :-

$-CH_2-OH$ Primary (1°)

$>CH-OH$ Secondary (2°)

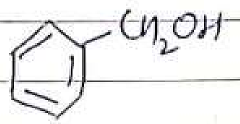
$-C-OH$ Tertiary (3°)

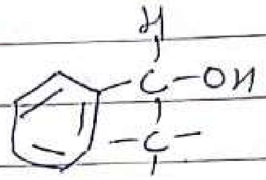
(b) Allylic alcohols :-

$CH_2=CH-CH_2-OH$

$CH_2=CH-C(H)(-C-)-OH$

(c) Benzylic alcohol

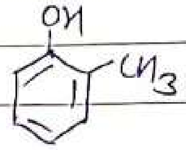




(ii) Compounds containing $C_{sp^2}-OH$ bond :-

(a) Vinylic alcohol: $CH_2=CH-OH$

(b) Phenols :



2. Ethers :-

(i) Simple or symmetrical

e.g. Diethyl ether $C_2H_5OC_2H_5$

(ii) Mixed or Unsymmetrical

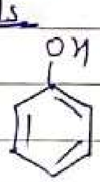
e.g. $C_2H_5OCH_3$ ethylmethyl ether

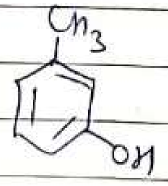
NOMENCLATURE

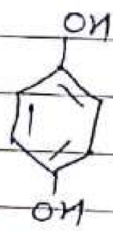
(a) Alcohols :-

	CH_3OH	$\underset{OH}{CH_2}-\underset{OH}{CH}-\underset{OH}{CH_2}$
Common	Methyl alcohol	Glycerol
IUPAC	Methanol	Propane-1,2,3-triol

(b) Phenols







Common	Phenol	m-Cresol	Quinol
IUPAC	Phenol	3-Methylphenol	Benzene-1,4-diol

(c) Ethers :-

	CH_3OCH_3	$C_6H_5OCH_3$	$C_2H_5OC_2H_5$
Common	Dimethyl ether	Methyl phenyl ether	Diethyl ether
IUPAC	Methoxymethane	Methoxybenzene	Ethoxyethane

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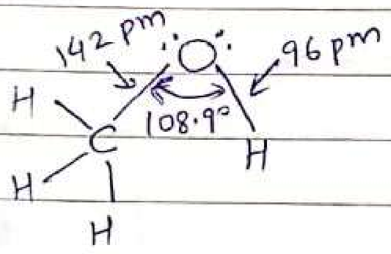
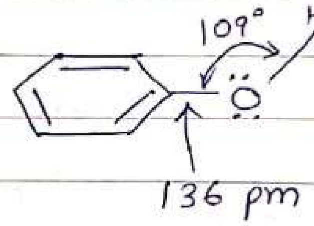
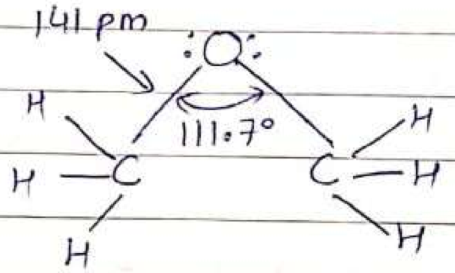
STRUCTURES OF FUNCTIONAL GROUPS



$\longrightarrow$ Methanol

Phenol $\longleftarrow$ 

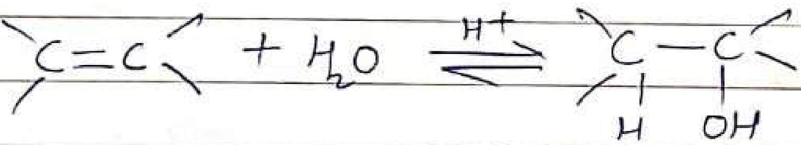


$\longrightarrow$ Methoxymethane

PREPARATION OF ALCOHOLS

1. From Alkenes :-

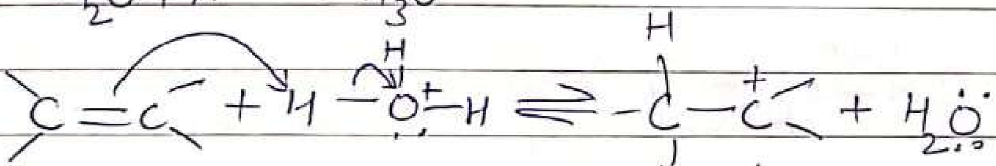
(i) By acid catalysed hydration :



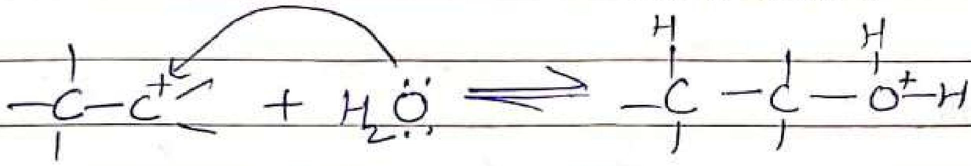
$$CH_3CH=CH_2 + H_2O \underset{}{\overset{H^+}{\rightleftharpoons}} CH_3-\underset{OH}{\underset{|}{CH}}-CH_3$$

Mechanism :-

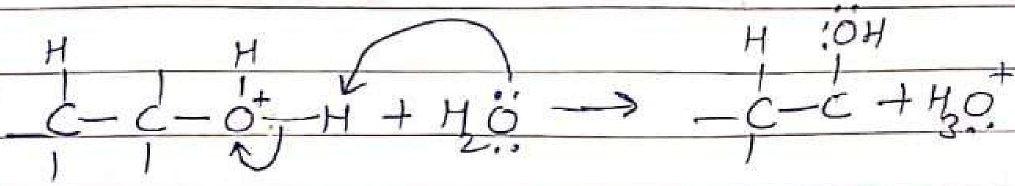
Step 1 :- $H_2O + H^+ \longrightarrow H_3O^+$



Step 2 :-



Step 3 :-



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(ii) By hydroboration-oxidation

$$CH_3-CH=CH_2 + \underset{\text{Diborane}}{(H-BH_2)_2} \longrightarrow CH_3-\underset{H}{\underset{|}{CH}}-\underset{BH_2}{\underset{|}{CH_2}}$$

$$\downarrow CH_3-CH=CH_2$$

$$(CH_3-CH_2-CH_2)_3B \xleftarrow{CH_3-CH=CH_2} (CH_3-CH_2-CH_2)_2BH$$

$$H_2O \downarrow 3H_2O_2, \bar{O}H$$

$$3CH_3-CH_2-CH_2-OH + B(OH)_3$$

Propan-1-ol

2. From Carbonyl ($>C=O$) compounds :-

(i) By reduction of aldehydes and ketones :-

$$RCHO + H_2 \xrightarrow{Pt/Pd/Ni} RCH_2OH \quad (1°)$$

$$RCOR' \xrightarrow{NaBH_4/LiAlH_4} R-\underset{OH}{\underset{|}{CH}}-R' \quad (2°)$$

(ii) By reduction of carboxylic acids and esters :-

$$RCOOH \xrightarrow[\text{(ii) } H_2O]{\text{(i) } LiAlH_4} RCH_2OH$$

Commercially :-

$$RCOOH \xrightarrow[H^+]{R'OH} RCOOR' \xrightarrow[\text{Catalyst}]{H_2} RCH_2OH + R'OH$$

3. From Grignard reagents :-

Step 1:-

$$>C=O + \overset{\delta-}{R}\leftarrow\overset{\delta+}{Mg}-X \longrightarrow \underset{\text{Adduct}}{\left[>\underset{R}{\underset{|}{C}}-O^-\overset{+}{Mg}-X\right]}$$

$$>\underset{R}{\underset{|}{C}}-OH + Mg(OH)X \xleftarrow{H_2O}$$

step 2

$$HCHO + RMgX \longrightarrow RCH_2OMgX \xrightarrow{H_2O} RCH_2OH + Mg(OH)X$$

Methanal $\qquad\qquad\qquad\qquad\qquad\qquad$ 1° alcohol

$$RCHO + R'MgX \longrightarrow R-\overset{R'}{\overset{|}{C}}H-OMgX \xrightarrow{H_2O} R-\overset{R'}{\overset{|}{C}}H-OH + Mg(OH)X$$

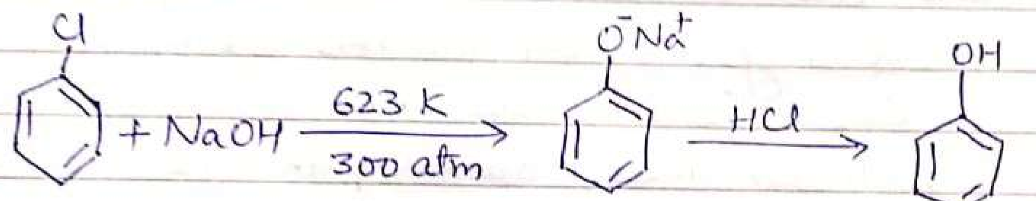
Aldehyde $\qquad\qquad\qquad\qquad\qquad\qquad$ 2° alcohol

$$RCOR + R'MgX \longrightarrow R-\underset{R}{\underset{|}{\overset{R'}{\overset{|}{C}}}}-OMgX \xrightarrow{H_2O} R-\underset{R}{\underset{|}{\overset{R'}{\overset{|}{C}}}}-OH + Mg(OH)X$$

Ketone $\qquad\qquad\qquad\qquad\qquad\qquad$ 3° alcohol

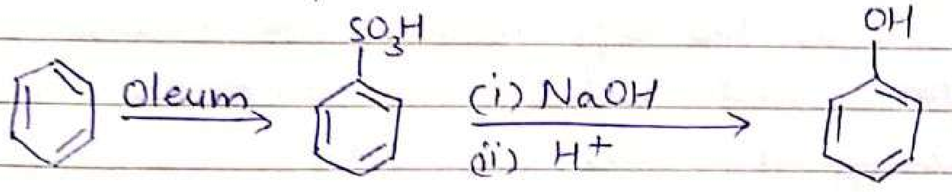
PREPARATION OF PHENOLS :-

1. From haloarenes :-

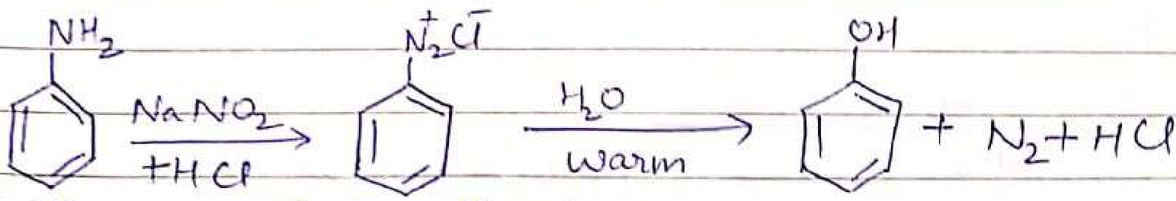


sodium phenoxide $\qquad$ Phenol

2. From benzenesulphonic acid :-

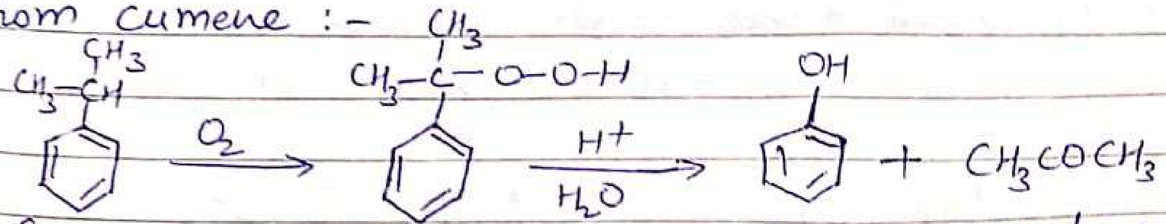


3. From diazonium salts :-



Aniline $\qquad$ Benzene diazonium chloride

4. From cumene :-



Cumene (isopropylbenzene) $\qquad$ Cumene hydroperoxide $\qquad$ Phenol $\qquad$ acetone

PHYSICAL PROPERTIES :-

ALCOHOLS :- (i) Lower members are colourless liquids with distinct smell and burning taste, while higher members are colourless, odourless waxy solids.

(ii) Boiling point increase with increase of no. of carbon atoms due to increase in van der Waals forces.

Among the isomeric alcohols the order of b.p.

1° Alcohol > 2° alcohol > 3° alcohol

B.p. of alcohols are higher in comparison to other classes of compounds with comparable mass due to the presence of intermolecular hydrogen bonding.

(iii) Solubility :- Lower members are highly soluble in water. Solubility decreases with the increase of molecular mass. Solubility in water is due to H-bonding. Order of solubility

Branched chain alcohol > straight chain alcohol

PHENOLS :- (i) Colourless, crystalline solids

(ii) In phenols too b.p. increase with increase of C-atoms and higher in comparison to other classes of compounds.

(iii) Moderately soluble in cold H_2O but readily soluble in ethanol and ether.

CHEMICAL REACTIONS :-

(i) Alcohols as nucleophiles

$$R-\ddot{O}-H + \overset{+}{C}- \longrightarrow R-\overset{H}{\overset{+}{O}}-\overset{|}{\underset{|}{C}}- \longrightarrow R-O-\overset{|}{\underset{|}{C}}- + H^+$$

(ii) Protonated alcohols as electrophiles :-

$$R-CH_2-OH + H^+ \longrightarrow R-CH_2-\overset{+}{O}H_2$$

$$Br^- + \underset{R}{\underset{|}{CH_2}}-\overset{+}{O}H_2 \longrightarrow Br-\underset{R}{\underset{|}{CH_2}} + H_2O$$

(a) Reactions involving cleavage of O–H bond

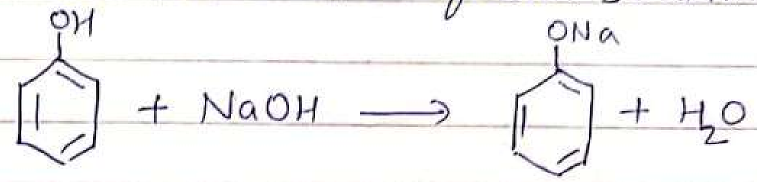
1. Acidity of alcohols and phenols

(i) Reaction with metals

Alcohols/Phenols + Active metals $\longrightarrow$ alkoxides or Phenoxides + H_2

e.g. $2R-OH + 2Na \longrightarrow 2R-O-Na + H_2$
Sodium alkoxide

Phenol also react with aqueous sodium hydroxide.



They are Bronsted acids

$$B: + H-\ddot{O}-R \longrightarrow B-H + :\ddot{O}^{-}-R$$

Base, Acid, Conjugate acid, Conjugate base

(ii) Acidity of alcohols :-

Acidic strength of alcohols decreases

1° Alcohol > 2° alcohol >> 3° alcohol

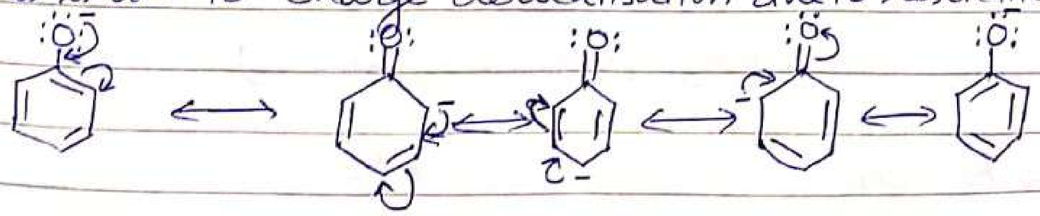
Alcohols are weaker acids than water.

Alcohols act as Bronsted bases as well.

(iii) Acidity of phenols :-

Phenols are stronger acids than alcohols and H_2O.

Phenol molecule is less stable than phenoxide ion, this is due to charge delocalisation due to resonance.



Presence of electron withdrawing group (e.g. NO_2), enhances acidic strength of phenol (specially if at o- and p- positions.

Electron releasing groups decrease acid strength.

PHENOL is million times more acidic than ETHANOL.

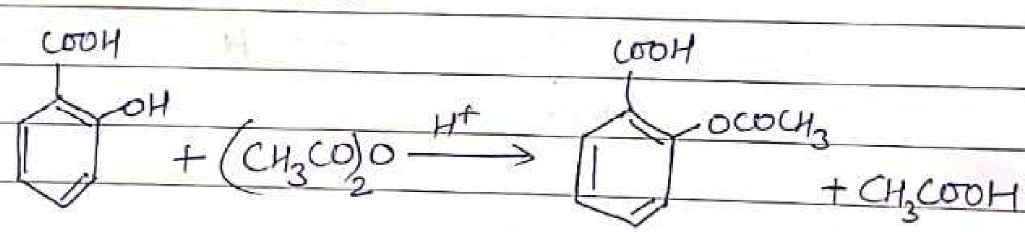
2. Esterification

$$\text{Alcohols/Phenols} + \text{Carboxylic acid/ Acid chlorides/ Acid anhydrides} \longrightarrow \underline{\text{Esters}}$$

$$\underset{}{Ar/ROH} + \underset{\text{Carboxylic acid}}{R'COOH} \overset{H^+}{\rightleftharpoons} Ar/RCOOR' + H_2O$$

$$Ar/ROH + \underset{\text{Acid anhydride}}{(R'CO)_2O} \overset{H^+}{\rightleftharpoons} Ar/ROCOR' + R'COOH$$

$$R/ArOH + \underset{\text{Acid chloride}}{R'COCl} \xrightarrow{\text{Pyridine}} R/ArOCOR' + HCl$$



Salicylic acid

Acetylsalicylic acid (Aspirin

(b) Reaction involving cleavage of C—O bond :

Only alcohols show these reactions while phenols show this only with Zn.

1. Reaction with hydrogen halide :

$$ROH + HX \longrightarrow RX + H_2O$$

2. Reaction with PX_3 (phosphorus trihalide)

$$3\,R{-}OH + PX_3 \longrightarrow 3R{-}X + H_3PO_3 \qquad (X = Cl, Br)$$

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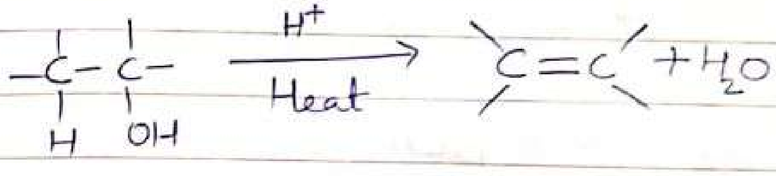
3. Dehydration



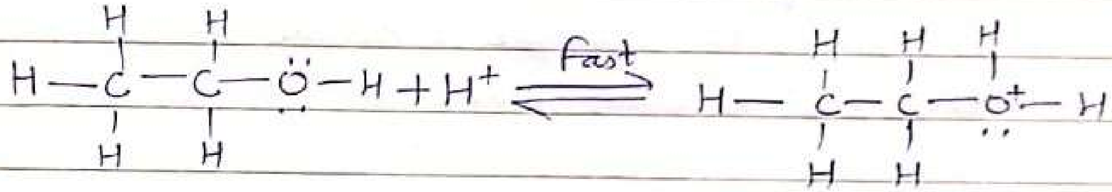
$$C_2H_5OH \xrightarrow[443\,K]{H_2SO_4} CH_2=CH_2 + H_2O$$

The relative ease of dehydration of alcohols

$$3° > 2° > 1°$$

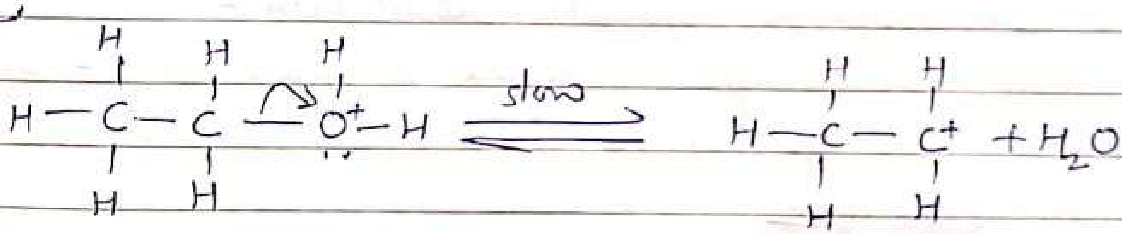
Mechanism :-

Step 1 :



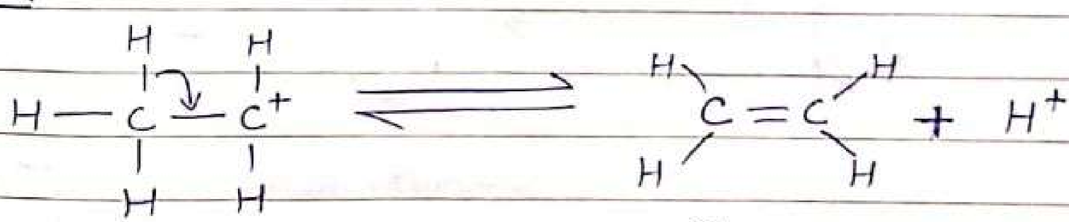
Protonated alcohol

Step 2 ;



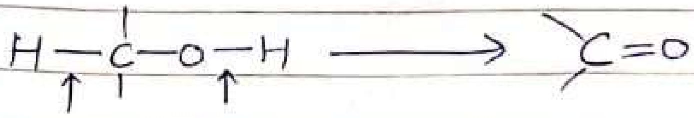
Carbocation

Step 3 :



Ethene

4. Oxidation



bond breaking Dehydrogenation

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$$RCH_2OH \xrightarrow{\text{Oxidation}} R-\overset{H}{\overset{|}{C}}=O \longrightarrow R-\overset{OH}{\overset{|}{C}}=O$$

$$RCH_2OH \xrightarrow{CrO_3} RCHO$$

PCC (Pyridinium chlorochromate) also gives a good yield of aldehyde from 1° alcohols.

$$\underset{\text{Sec-alcohol}}{R-\underset{OH}{\underset{|}{CH}}-R'} \xrightarrow{CrO_3} \underset{\text{Ketone}}{R-\underset{O}{\underset{||}{C}}-R'}$$

Tertiary alcohols donot oxidize.

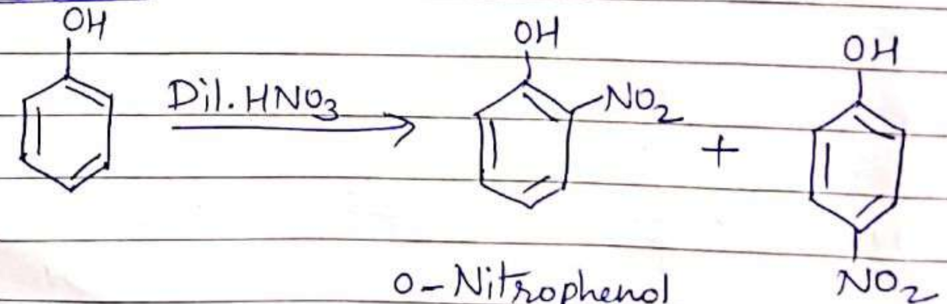
Vapours of 1° and 2° alcohols over heated Cu at 573 K gives aldehyde or ketone by dehydrogenation while 3° alcohols undergo dehydration.

(C) Reactions of phenols :-

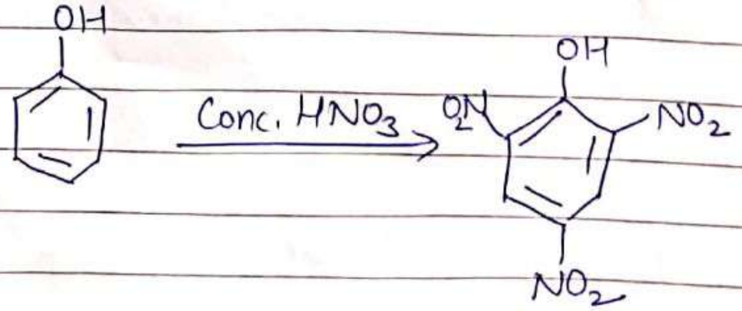
1. Electrophilic aromatic substitution :-

Incoming groups are directed to o- and p- positions as they are electron rich due to resonance.

(i) Nitration :



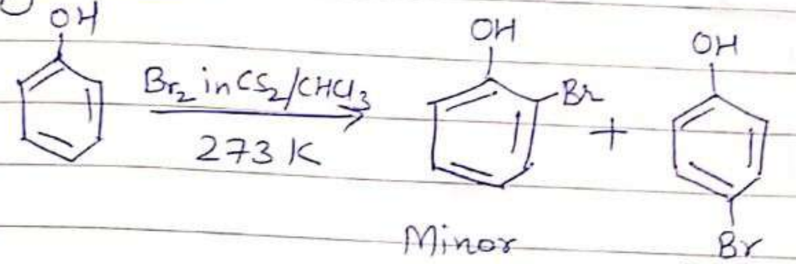
o-Nitrophenol p-Nitrophenol



2,4,6-Trinitrophenol
(Picric acid)

(ii) Halogenation :-

(a)

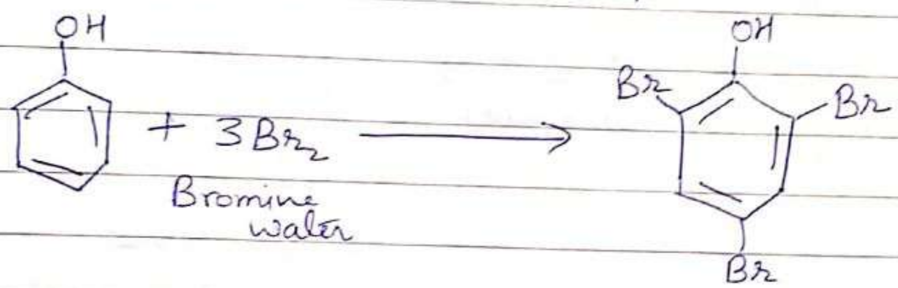


Minor

Major

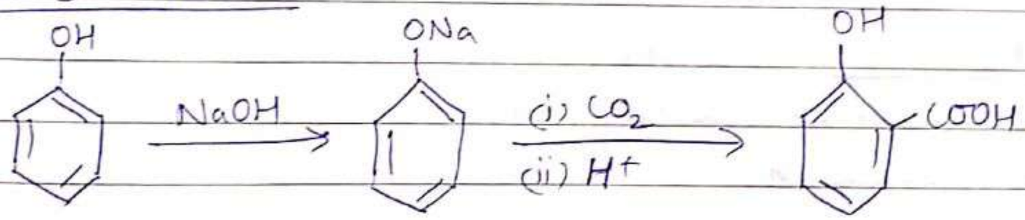
Monobromophenols

(b)



2,4,6-Tribromophenol
(white ppt)

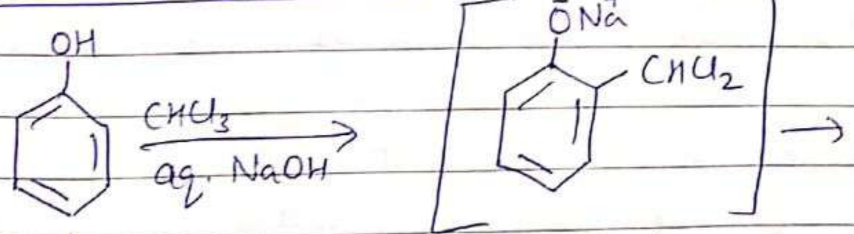
2. Kolbe's reaction :



Sodium Phenoxide

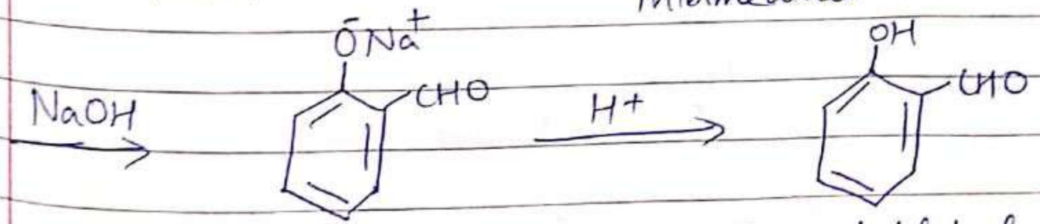
2-Hydroxybenzoic acid
(Salicylic acid)

3. Reimer-Tiemann reaction :-



Phenol

intermediate



Salicylaldehyde

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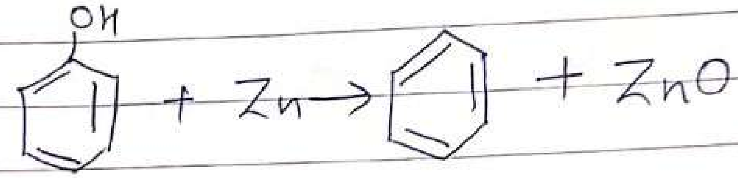
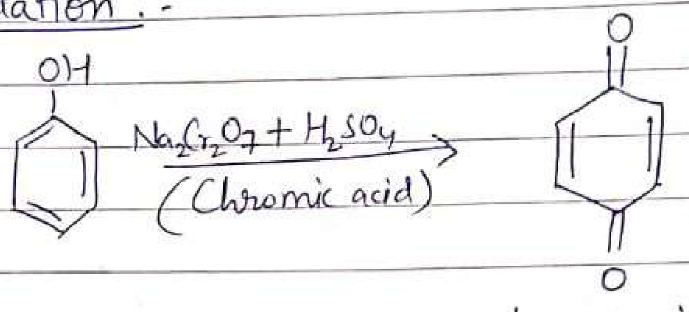
4. <u>With Zn dust</u> :-



5. <u>Oxidation</u> :-



benzoquinone
(a conjugated diketone)

SOME COMMERCIALLY IMPORTANT ALCOHOLS

1. <u>Methanol</u> (CH_3OH)
   * known as wood spirit
   * Preparation :-

$$CO + 2H_2 \xrightarrow[\substack{200\text{-}300\ atm \\ 573\text{-}673\ K}]{ZnO\text{-}Cr_2O_3} CH_3OH$$

* Colourless liquid, B.P. 337 K
* Poisonous, even cause death if ingested in large amount.
* Use - (i) Solvent in paint, varnishes
  (ii) for making formaldehyde.

2. <u>Ethanol</u> (C_2H_5OH)
   * Obtained by fermentation

$$\underset{\text{Sugar(fruits)}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{\text{Invertase}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

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$$C_6H_{12}O_6 \xrightarrow{\text{Zymase (Yeast)}} 2C_2H_5OH + 2CO_2$$

* Colourless liquid, B.P. 351 K
* Use – (i) Solvent in paint industry
  (ii) For preparing a no. of carbon compounds
* $CuSO_4$ (for colour) and pyridine (for foul smell) are mixed to make commercial alcohol unfit for drinking. (Denaturation of alcohol)

USES OF ALCOHOLS AND PHENOLS

- Alcohols are used as solvents, anti-freeze agent, in the preparation of medicines, as preservatives etc.
- A mixture of 20% ethanol and 80% gasoline is the power alcohol.
- Phenols are used to prepare bakelite, plastic, for manufacturing of dyes and drugs, in medicines, in the preparation of phenolphthalein etc.

"ETHERS"

Ethers are represented as R-OR' and have general formula $C_nH_{2n+2}O$.
Ethers are dialkyl derivatives of water or monoalkyl derivatives of alcohols.

Preparation of Ethers :-

1. By dehydration of alcohols :-

$$CH_3CH_2OH \xrightarrow[443\,K]{H_2SO_4} CH_3{=}CH_2$$

$$CH_3CH_2OH \xrightarrow[413\,K]{H_2SO_4} C_2H_5OC_2H_5$$

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Ether is formed by S_N2 reaction as below:

(i) $CH_3-CH_2-\ddot{O}-H + H^+ \longrightarrow CH_3-CH_2-\overset{+}{\ddot{O}}(H)-H$

(ii) $CH_3-CH_2-\ddot{O}(H): + CH_3-CH_2-\overset{+}{O}(H)-H \longrightarrow CH_3CH_2-\overset{+}{O}(H)-CH_2-CH_3 + H_2O$

(iii) $CH_3-CH_2-\overset{+}{O}(H)-CH_2CH_3 \longrightarrow CH_3CH_2-O-CH_2CH_3 + H^+$

2. Williamson synthesis :- Imp. Laboratory method. Both symmetrical and unsymmetrical ethers can be prepared.

$$R-X + R'-\ddot{\bar{O}}\overset{+}{Na} \longrightarrow R-\ddot{O}-R' + NaX$$

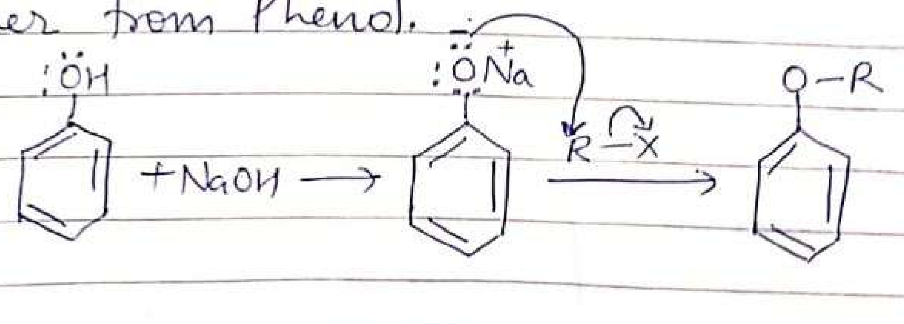
$$CH_3-C(CH_3)_2-\ddot{\bar{O}}Na^+ + CH_3-Br \longrightarrow CH_3-\ddot{O}-C(CH_3)_2-CH_3 + NaBr$$

If 3° alkyl halide is used, alkene is produced.

$$CH_3-C(CH_3)_2-Br + \overset{+}{Na}\ddot{\bar{O}}-CH_3 \longrightarrow CH_3-C(CH_3)=CH_2 + CH_3OH$$

2-Methylpropene

Ether from Phenol.



Physical properties of Ether :-

* They have net dipole moment as C–O bond is polar.
* B.P. is less with alcohols with comparable mass as alcohols have hydrogen bonding.
* Miscibility with water to same extent as alcohols.

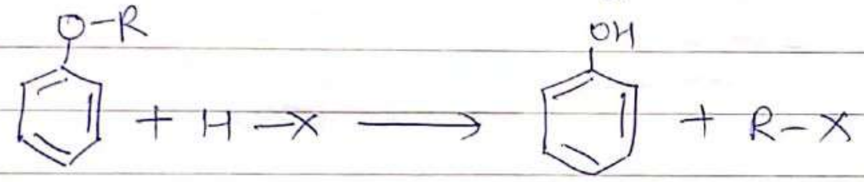
CHEMICAL REACTIONS :-

1. Cleavage of C–O bond in ethers :-

They are least reactive, under drastic conditions dialkyl ether gives 2 RX molecules.

$$R-O-R + HX \longrightarrow RX + R-OH$$

$$R-OH + HX \longrightarrow RX + H_2O$$



Alkyl aryl ether. Phenol alkyl halide

Order of reactivity of HX :

$$HI > HBr > HCl$$

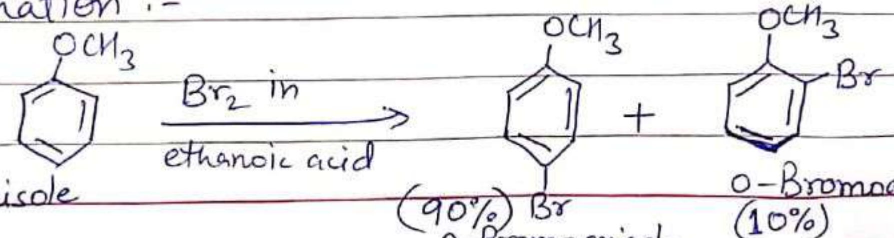
$$CH_3-\underset{CH_3}{\overset{CH_3}{C}}-O-CH_3 + HI \longrightarrow CH_3OH + CH_3-\underset{CH_3}{\overset{CH_3}{C}}-I$$

3° alkyl group 3° halide

2. Electrophilic substitution :-

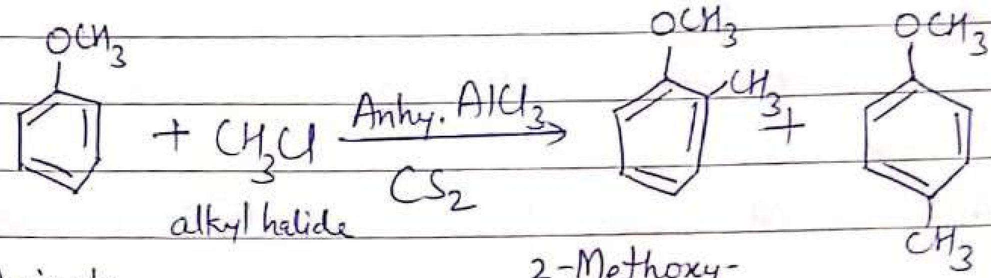
The alkoxy group (–OR) is o–, p– directing

(i) Halogenation :-



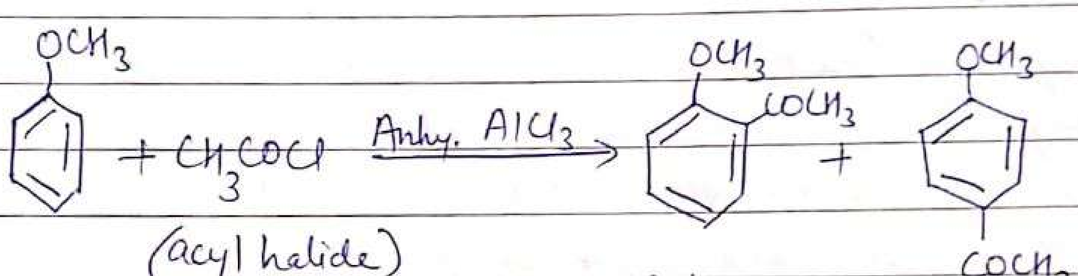
Anisole (90%) p-Bromoanisole o-Bromoanisole (10%)

(ii) Friedel-Crafts reaction :-



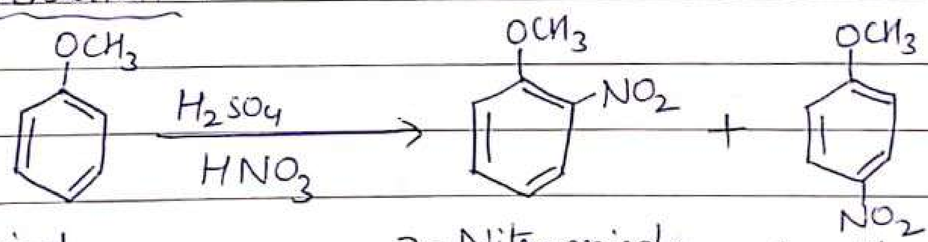
alkyl halide

Anisole — 2-Methoxy-toluene (Minor) — 4-Methoxytoluene (Major)



(acyl halide) Ethanoyl Chloride — 2-Methoxy-acetophenone (Minor) — 4-Methoxy-acetophenone (Major)

(iii) Nitration :-

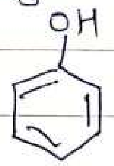


Anisole — 2-Nitroanisole (Minor) — 4-Nitroanisole (Major)

Use :- Ethers are inert and thus are used as solvents in many reactions.

Finally :- " Alcohols, phenols and ethers are the basic compounds for the formation of detergents, antiseptics and fragrances, respectively."

Assignment

1. How are primary, secondary and tertiary alcohols prepared from Grignard reagents?
2. Write the equations involved in the following reactions:
   (i) Reimer-Tiemann reaction
   (ii) Williamson synthesis
   (iii) Kolbe's reaction
3. Write the major product in the following equations:-
   (i) $CH_3-CH_2OH \xrightarrow{PCl_5} ?$
   (ii)  $+ CH_3-Cl \xrightarrow{anhy.AlCl_3} ?$
   (iii) $CH_3-Cl + CH_3CH_2-ONa \longrightarrow ?$
4. Give reason for
   (a) Phenol is a stronger acid than alcohol.
   (b) The boiling point of ethanol is higher than that of methanol.
   (c) Alcohols are comparatively more soluble in water than the corresponding hydrocarbons.
5. Describe the mechanism of alcohols reacting both as nucleophiles and as electrophiles in their reactions.
6. How would you convert -
   (i) Phenol to benzoquinone?
   (ii) Propene to propan-2-ol?
   (iii) Benzyl chloride to benzyl alcohol?
7. Give mechanism of preparation of ethoxyethane from ethanol.

"Believe in the Power of Positivity." STAY "HOME SAFE"

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CBSE Class 12 Biology

Revision Notes

Chapter-03 Human Reproduction

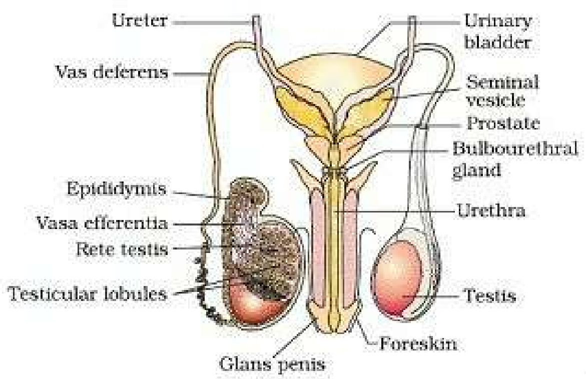
Video Watch : https://youtu.be/Lbv6WbjIQW0

Humans are sexually reproducing and viviparous. The reproductive events in humans include formation of gametes (gametogenesis), i.e., sperms in males and ovum in females, transfer of sperms into the female genital tract (insemination) and fusion of male and female gametes (fertilisation) leading to formation of zygote. This is followed by formation and development of blastocyst and its attachment to the uterine wall (implantation), embryonic development (gestation) and delivery of the baby(parturition)

The Male Reproductive System: It consists of:

i. Primary sex organs i.e. a pair of testes suspended in ascrotum.
ii. Secondary sex organs i.e. a pair of ducts each differentiated into rete testis, vasa efferentia,epididymisandvasdeferens,ejaculatoryductandtheassociatedglands
iii. External genitalia

- The testes are situated outside the abdominal cavity in a pouch called **scrotum,** which help in maintaining the low temperature of testes necessary for spermatogenesis.
- Each testes has about 250 testicular lobules and each lobule contain highly coiled **seminiferous tubules** in which sperms are produced. Each seminiferous tubules is lined by two types of cells, **spermatogonia** (male germ cell) and **Sertoli cells**.
- **Leydig cells** or interstitial cells present around the seminiferous tubules synthesize and secrete androgen hormone.



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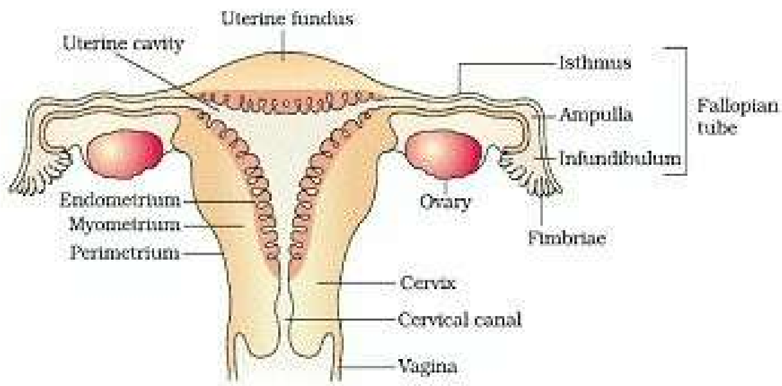
Ejaculatory duct store and transport the sperm from testes to outside through urethra which originate from urinary bladder and extend through penis to its external opening **urethral meatus.**

- The penis is male external genitalia. The enlarged end of penis is called the **glans penis** is covered by a loose fold of skin called **foreskin**.
- Male accessary glands include paired **seminal vesicles, prostrate** and **paired bulbourethralglands**.Secretionoftheseglandsformstheseminalplasmawhich containsfructose,calciumandenzymes.Thesecretionofbulbourethralglandsalso helps in lubrication of the penis.

The Female Reproductive System: It consists of :

a. The primary sex organ that is a pair ofovaries
b. Secondary sex organs- the duct system consisting of a pair of fallopian tube , a uterus , cervix andvagina
c. Externalgenitalia
d. Mammaryglands

- Ovaries are primary female sex organ that produce the female gamete and several steroid hormones. Each ovary is covered by thin epithelium which encloses the ovarian stroma, which is divided into a peripheral cortex and an inner medulla.
- Fallopiantubeextendsfromperipheryofovarytotheuterus.Thepartclosertoovary isafunnelshapedstructurecalled**infundibulum**havingfingerlikeprojectioncalled **fimbriae.**
- Infundibulumleadsto**ampulla**andjoinwithuteruswith**isthmus**.Uterusispear shaped structure also calledwomb.
- Uterusopenvaginathroughanarrowcervix.Thecavityofcervix**(cervicalcanal**) along with vagina forms the birthcanal.
- The wall of uterus has three layers of tissue:



I. Perimetrium- external membrane.
II. Myometrium – middle thick layer of smooth muscles which exhibit strong contraction during delivery ofbaby.
III. Endometrium-linetheuterinewallandundergocyclicchangesduringmenstrualcycle.

Female external genitalia includes

- Mons pubis – cushion of fatty tissues covered by skin and pubic
- hair. Labia majora- fleshy fold that surround the vaginal opening.
- Labia manora – paired fold of tissue under labia majora.
- The opening of vagina is often partially covered by a membrane called **hymen**. The tiny finger like projection present at the upper junction of two labia manora above the urethral opening is called **clitoris**.

Mammaryglandsarepairedstructuresthatcontainglandulartissuesandvariablefats.Each glandulartissuecontains15-20mammarylobescontainingalveolithatsecretemilk. Mammary ducts join to form mammary ampulla.

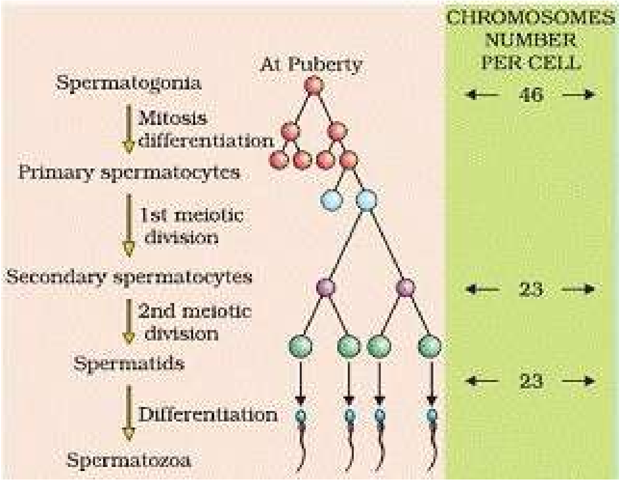
Gametogenesis: The process of formation of male and female gametes in testes and ovary respectively is called gametogenesis.It is of two types:

1. **Spermatogenesis** inmales
2. **Oogenesis** infemales

Spermatogenesis- in testes immature, male germ cells (spermatogonia) produce sperm by spermatogenesis that begin at puberty.

- The spermatogonia present at the inner side of seminiferous tubules multiply by mitotic division and increase in number. Each spematogonium contain 46 chromosomes.
- Spermatogonia forms spermatocyte that undergo meiotic division to reproduce secondary spermatocytes having 23 chromosomes.
- The spermatids are transformed into spermatozoa by the process called **spermiogenesis**. The sperm heads remain embedded in sertoli cells and are released from seminiferous tubules by the process of **spermiation.**

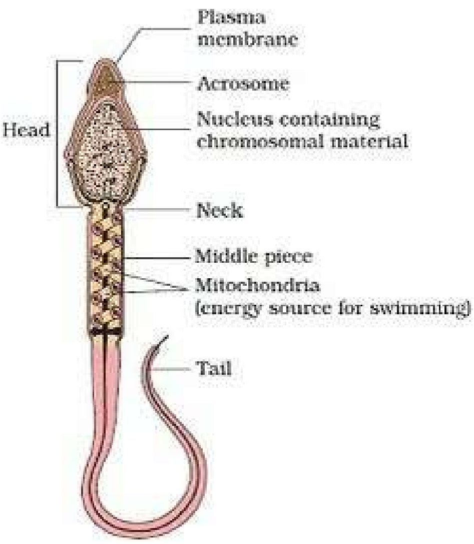
Materialdownloadedfrom myCBSEguide.com.



Hormonal control of spermatogenesis

- Spermatogenesis initiated due to increase in secretion of gonadotropin releasing hormone by hypothalamus
- Increase in GnRH act on anterior pituitary and stimulate secretion of two gonadotropins, LH and FSH
- LH acts on Leydig cells and stimulates them to secrete androgens.
- FSH acts on Sertoli cells, stimulates secretion of some factors which help in spermiogenesis

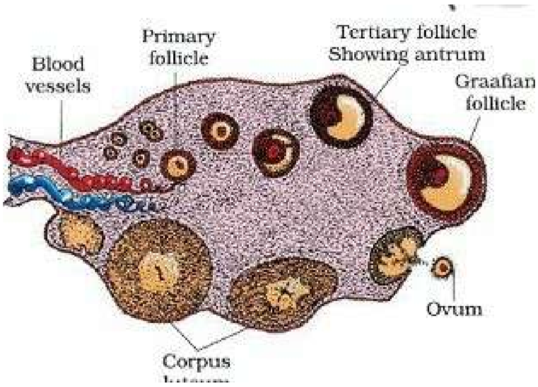
Structure of sperm- sperm is a microscopic structure composed of a **head, neck,** a **middle piece** and a **tail.** The sperm head contain elongated haploid nucleus, anterior portion of which is covered by cap like structure **acrosome**.



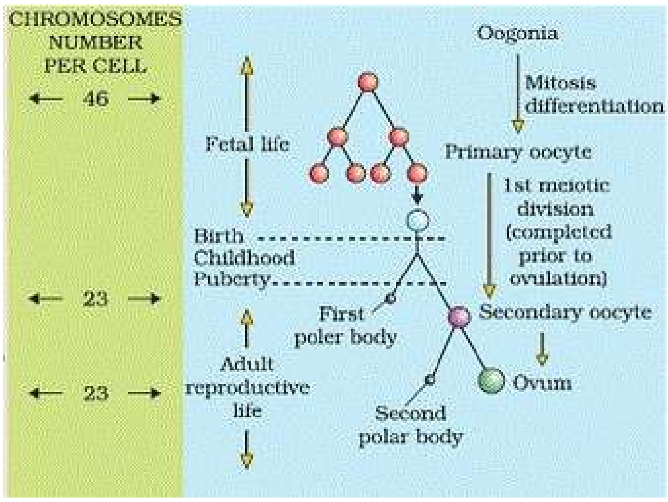
Human male ejaculates about 200-300 million sperms during a coitus. The seminal plasma along with the sperms constitutes the semen. The function of male sex secondary ducts and glands are maintained by androgen hormones.

Oogenesis : The process of formation of mature female gametes is called oogenesis. It started during embryonic development stage when millions of ogonia (gamete mother cells) are formed in each fetal ovary.

- The gametes mother cells start division and enter into prophase-I of meiotic division and get temporally arrested at that stage called **primary oocytes**.
- Each primary oocyteget surrounded by a layer of granulosa cell than it is called the **primary follicle**.
- At puberty, about 60,000- 80,000 primary follicles are left in each ovary.



- Primary follicle gets surrounded by more layers of granulosa cells called secondary follicle that transform into tertiary follicle that contain fluid filled cavity called **antrum.**



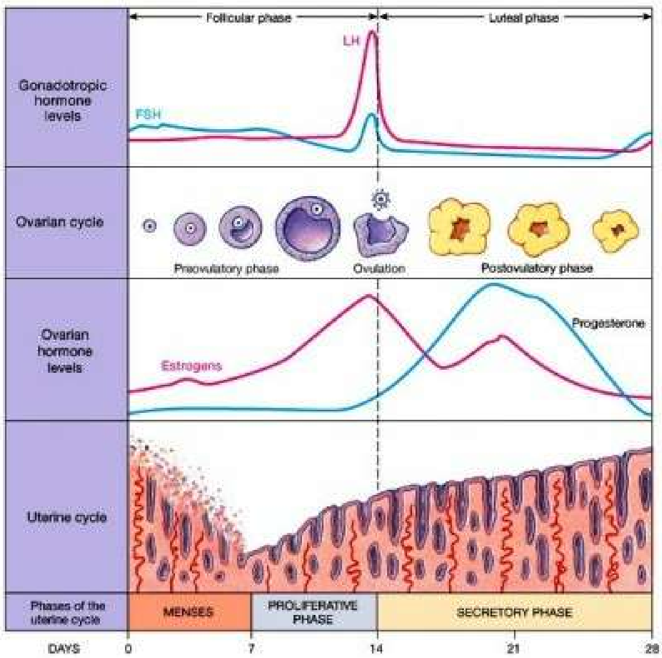
- The tertiary follicles further changes into the mature follicle called **Graafian follicle**, which rapture to release secondary oocytes (ovum) from the ovary by the process of ovulation.

Menstrual cycle: The reproductive cycles in female primates is called menstrual cycle. It start at puberty and is called **menarche**.

Phases of Menstrual Cycle

The menstrual cycle consists of following four phases:

1. **MenstrualPhase:**
    i. In a 28 days menstrual cycle,the menses takes place on cycle days3-5.
    ii. TheproductionofLHfromtheanteriorlobeofthepituitaryglandisreduced.
    iii. The withdrawal of this hormone causes degeneration of the corpus luteum and, therefore progestrone production isreduced.
    iv. Production of oestrogen is also reduced in thisphase.
    v. The endometrium of uterus breaks down & menstruationbegins.
    vi. The cells of endometrium secretions, blood & unfertilised ovum constitutes the menstrualflow.
2. **FollicularPhase:**
    i. Thisphaseusuallyincludescycledays6-13or14ina28dayscycle.
    ii. The follicle stimulating hormone (FSH) secreted by the anterior lobe of the pituitary gland stimulates the ovarian follicle to secreteoestrogens.
    iii. Oestrogen stimulates the proliferation of the endometrium of the uterinewall.
    iv. The endometrium becomes thicker by rapid cell multiplication and this is accompanied by an increase in uterine glands & bloodvessels.



3. **OvulatoryPhase:**

i. BothLH&FSHattainapeaklevelinthemiddleofcycle(about14thday).
ii. Oestrogen concentration in bloodincreases.
iii. RapidsecretionofLHinducesrupturingofgraffianfollicleandtherebythereleaseof ovum.
iv. In fact LH causesovulation.

4. **LutealPhase:**
   i. Includes cycle days 15 to28.
   ii. Corpus luteum secretesprogestrone.
   iii. Endometrium thickens.
   iv. Uterine glands becomesecretory.

Hormonal Control of MC

i. FSH stimulates the ovarian follicles to produceoestrogens.
ii. LH stimulates corpus luteum to secreteprogestrone.
iii. Menstrual phase is caused by the increased production ofoestrogens.
iv. LH causesovulation
v. Proliferative phase is caused by the increased production ofoestrogens.
vi. Secretory phase is caused by increased production ofprogestrone.

Fertilisation and Implantation

The process of fusion of sperm with ovum is called fertilisation.

- During coitus (copulation) semen is released into vagina. The motile sperms swim rapidly to reach the junction of isthmus and ampulla of fallopian tube. The ovum also reaches there and fusion of gametes takes place in at ampullary-isthmic junction.
- In this acrosome of sperm undergoes acrosomal reaction and releases certain sperm lysins which dissolve the egg envelopes locally and make the path for the penetration ofsperm.
- These sperm lysins contain a lysing enzyme hyaluronidase which dissolves the hyaluronic acid polymers in the intercellular spaces which holds the granulosa cells of corona radiata together; corona penetrating enzyme (that dissolves the corona radiata) and acrosin (which dissolves the zona pellucida). Then it dissolves the zona pellucida.

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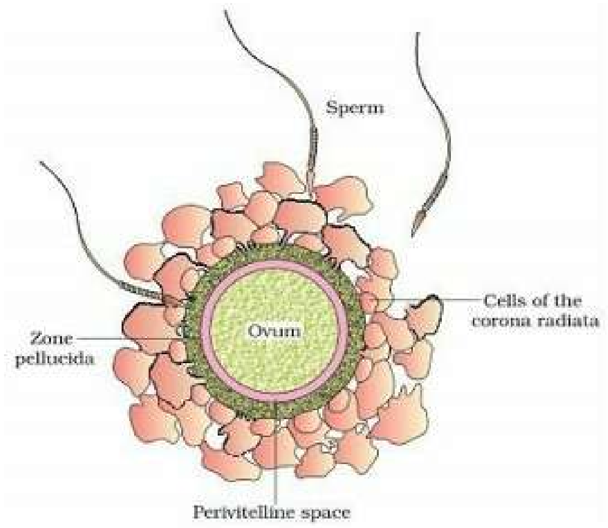
Cortical reaction:

1. (a) Immediately after the entry of a sperm into the egg, the later shows a cortical reaction to check the entry of moresperms.
2. (b)Inthisreaction,thecorticalgranulespresentbeneaththeegg’splasmamembrane releasechemicalsubstancebetweentheooplasmandtheplasmamembrane(vitellin e membrane).
3. (c)Thesesubstancesraisethevitellinemembraneabovetheeggsurface.Theelevate d vitelline membrane is called fertilizationmembrane.
4. (d) The increased space between the ooplasm and the fertilization membrane and the chemical present in it effectively check the entry of othersperm.
5. (e) If polyspermy occurs, that is more than one sperm enter the secondary oocyte, the resulting cell has too much genetic material to developnormally



- The haploid gametes fuse together to form diploid zygote. As the zygote moves towards the uterus, the mitotic division starts and form cleavage to change into 2, 4,8,16 celled blastomeres.
- The blastomeres with 8 to 16 cells are called morula. Morula divide to change into blastocysts .The blastomeres in the blastocyst are arranged into an outer layer called **trophoblast** and an inner group of cells attached to trophoblast called the **inner cell mass**.The outer layer of blastocyst is called trophoblast that attach with endometrium of uterus, called **implantation** that leads to pregnancy.

Pregnancy and embryonic development

The finger-like projections on trophoblaste after implantation called is called **chronic villi** that along with uterine wall forms functional unit between developing embryo and maternal body called **placenta**. Placenta is attached with fetus with an umbilical cord that transport food and oxygen toembryo.

- Hormones**hCG(humanchorionicgonadotropin),hPL(humanplacentallactogen) and relaxin** are produced in woman only during pregnancy byplacenta.
- After implantation, the inner cell mass (embryo) differentiates into an outer layer called **ectoderm** and an inner layer called **endoderm**. A **mesoderm** soon appears between the ectoderm and the endoderm. These three layers give rise to all tissues (organs) in adults. It is important to note that the inner cell mass contains certain cells calledstemcellswhichhavethepotencytogiverisetoallthetissuesandorgans
- Inhuman,afteronemonthofpregnancytheembryo'sheartisformed.Bytheendof 2^{nd}monthlimbsanddigitsareformed.Bytheendof12months,majororgansand externalgenitalorgansarewelldeveloped.Thefirstmovementoffoetusisobserved in 5 months. By the end of 24 weeks body is covered with fine hair, eye lids and eyelessareformed.Attheendof9monthsfetusisfullydeveloped.

PARTURITION AND LACTATION

Parturition-the process of delivery of fully developed foetus is calledparturition.

- Signals for parturition originate from the fully developed fetus and placenta inducing mild uterine contractions called **Foetal ejection reflex**
- It triggers the release of oxytocin from maternal pituitary

The mammary glandsof female, start producing milk, to the end of pregnancy by the process of **lactation**. The milk produced during the initial few days of lactation is called **colostrum**, which contain several antibodies.

WEEK-4 (H.H.W)

ST. MARY'S PUBLIC SCHOOL

Mathematics

CLASS- XII

SOLUTIONS

OF

N.C.E.R.T

(EX.4.1 TO EX.4.6)

INCLUDING SELF EVALUATION TEST(10 MARKS)

CHAPTER

DETERMINANTS

(DO IN THE REGISTER: 30+10 MARKS)

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Mathematics

(Chapter - 4) (Determinants)
(Class 12)

Exercise 4.1

Evaluate the determinants in Exercises 1 and 2.

Question 1:

$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

Answer 1:

$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$ Expanding along R_1, we get

$= 2 \times (-1) - 4 \times (-5) = -2 + 20 = 18$

Question 2:

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ **(ii)** $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

Answer 2:

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos\theta \times \cos\theta - \sin\theta \times (-\sin\theta) = \cos^2\theta + \sin^2\theta = 1$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = (x^2 - x + 1) \times (x + 1) - (x - 1) \times (x + 1)$

$= x^3 + x^2 - x^2 - x + x + 1 - (x^2 + x - x - 1)$

$= x^3 - x^2 + 2$

Question 3:

If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Answer 3:

$|2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$ Expanding along R_1, we get

$= 2 \times 4 - 4 \times 8 = 8 - 32 = -24$ ... (1)

$4|A| = 4\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$ Expanding along R_1, we get

$= 4(1 \times 2 - 2 \times 4) = 4(-6) = -24$ ... (2)

From the equation (1) and (2), we get, $|2A| = 4|A|$

Question 4:

If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$

Answer 4:

$|3A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix}$ Expanding along R_1, we get

$= 3(36 - 0) - 0(0 - 0) + 1(0 - 0) = 108$ ... (1)

$27|A| = 27\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$ Expanding along R_1, we get

$= 27\{1(4 - 0) - 0(0 - 0) + 1(0 - 0)\} = 27(4) = 108$ ... (2)

From the equation (1) and (2), we get, $|3A| = 27|A|$

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Question 5:

Evaluate the determinants:

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ **(ii)** $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ **(iii)** $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ **(iv)** $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Answer 5:

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ Expanding along R_1, we get

$= 3(0-5) + 1(0+3) - 2(0-0) = -15 + 3 - 0 = -12$

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ Expanding along R_1, we get

$= 3(1+6) + 4(1+4) + 5(3-2) = 21 + 20 + 5 = 46$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ Expanding along R_1, we get

$= 0(0+9) - 1(0-6) + 2(-3-0) = 0 + 6 - 6 = 0$

(iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ Expanding along R_1, we get

$= 2(0-5) + 1(0+3) - 2(0-6) = -10 + 3 + 12 = 5$

Question 6:

If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find $|A|$.

Answer 6:

$|A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$ Expanding along R_1, we get

$= 1(-9+12) - 1(-18+15) - 2(8-5) = 3 + 3 - 6 = 0$

Question 7:

Find values of x, if

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ **(ii)** $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Answer 7:

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2 - 20 = 2x^2 - 24$ $\quad \Rightarrow x^2 = 3$ $\quad \Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 10 - 12 = 5x - 6x$ $\quad \Rightarrow -2 = -x$ $\quad \Rightarrow x = 2$

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Question 8:

If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to:

(A) 6 (B) ± 6 (C) -6 (D) 0

Answer 8:

$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

Hence, the option (B) is correct.

Exercise 4.2

Using the property of determinants and without expanding in Exercises 1 to 7, prove that:

Question 1:

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$$

Answer 1:

$$\text{LHS} = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = \begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix} \qquad [\text{Applying } C_1 \to C_1 + C_2]$$

$$= 0 = \text{RHS} \qquad [\because\ C_1 = C_3\ ]$$

Question 2:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

Answer 2:

$$\text{LHS} = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix} \qquad [\text{Applying } C_1 \to C_1 + C_2 + C_3]$$

$$= 0 = \text{RHS} \qquad [\because \text{ In column } C_1 \text{ every element is zero.}]$$

Question 3:

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$$

Answer 3:

$$\text{LHS} = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix} \qquad [\text{Applying } C_3 \to C_3 - C_1]$$

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$$= 9\begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix}$$ [Taking common 9 from C_3]

$= 0 =$ RHS [$\because\ C_2 = C_3$]

Question 4:

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

Answer 4:

$$\text{LHS} = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = \begin{vmatrix} 1 & bc & ab+bc+ca \\ 1 & ca & ab+bc+ca \\ 1 & ab & ab+bc+ca \end{vmatrix}$$ [Applying $C_3 \to C_3 + C_2$]

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$$= (ab + bc + ca)\begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix} \quad \text{[Taking } ab + bc + ca \text{ as common from } C_3]$$

$$= 0 = \text{RHS} \quad [\because C_1 = C_3]$$

Question 5:

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Answer 5:

$$\text{LHS} = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= \begin{vmatrix} 2c & 2r & 2z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} \quad [\text{Applying } R_1 \to R_1 + R_2 - R_3]$$

$$= 2\begin{vmatrix} c & r & z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} \quad [\text{Taking 2 as common from } R_1]$$

$$= 2\begin{vmatrix} c & r & z \\ a & p & x \\ a+b & p+q & x+y \end{vmatrix} \quad [\text{Applying } R_2 \to R_2 - R_1]$$

$$= 2\begin{vmatrix} c & r & z \\ a & p & x \\ b & q & y \end{vmatrix} \quad [\text{Applying } R_3 \to R_3 - R_2]$$

$$= -2\begin{vmatrix} a & p & x \\ c & r & z \\ b & q & y \end{vmatrix} \quad [\text{Applying } R_1 \leftrightarrow R_2]$$

$$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \text{RHS} \quad [\text{Applying } R_2 \leftrightarrow R_3]$$

Q.NO.6 TO Q.NO. 9 TRY YOURSELF

[FOR SOLUTION WATCH MY VIDEO LESSON]

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Question 10:

(i) $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

(ii) $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

Answer 10:

(i) LHS $= \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

$= \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$ [Applying $C_1 \to C_1 + C_2 + C_3$]

$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$ [Taking $5x+4$ as common from C_1]

$= (5x+4)\begin{vmatrix} 0 & x-4 & 0 \\ 0 & 4-x & x-4 \\ 1 & 2x & x+4 \end{vmatrix}$ [Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$]

$= (5x+4)\{(x-4)(x-4) - (4-x)0\}$ [Expanding along C_1]

$= (5x+4)(4-x)^2 =$ RHS

(ii) LHS $= \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$

$= \begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix}$ [Applying $C_1 \to C_1 + C_2 + C_3$]

$= (3y+k)\begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$ [Taking $3y+k$ as common from C_1]

$= (3y+k)\begin{vmatrix} 0 & -k & 0 \\ 0 & k & -k \\ 1 & y & y+k \end{vmatrix}$ [Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$]

$= (3y+k)\{(-k)(-k) - (k)0\}$ [Expanding along C_1]

$= (3y+k)k^2 =$ RHS

Question 11:

(i) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

(ii) $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

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Answer 11:

(i) LHS $= \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ [Applying $R_1 \to R_1 + R_2 + R_3$]

$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ [Taking $a+b+c$ as common from R_1]

$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ a+b+c & -a-b-c & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix}$ [By $C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$]

$= (a+b+c)\{(a+b+c)^2 - 0\}$ [Expanding along R_1]

$= (a+b+c)^3 =$ RHS

(ii) LHS $= \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$

$= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$ [Applying $C_1 \to C_1 + C_2 + C_3$]

$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$ [Taking $2(x+y+z)$ common from C_1]

$= 2(x+y+z)\begin{vmatrix} 0 & -(x+y+z) & 0 \\ 0 & x+y+z & -(x+y+z) \\ 1 & x & z+x+2y \end{vmatrix}$ [By $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$]

$= 2(x+y+z)\{(x+y+z)^2 - 0\}$ [Expanding along C_1]

$= 2(x+y+z)^3 =$ RHS

Question 12:

$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

Answer 12:

LHS $= \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix}$ [Applying $C_1 \to C_1 + C_2 + C_3$]

$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$ [Taking $1+x+x^2$ as common from C_1]

$= (1+x+x^2)\begin{vmatrix} 0 & x-1 & x^2-x \\ 0 & 1-x^2 & x-1 \\ 1 & x^2 & 1 \end{vmatrix}$ [Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$]

$= (1+x+x^2)(1-x)^2\begin{vmatrix} 0 & -1 & -x \\ 0 & 1+x & -1 \\ 1 & x^2 & 1 \end{vmatrix}$ [Taking $1-x$ as common from R_1 and R_2]

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$= (1+x+x^2)(1-x)^2\{1+x(1+x)\}$ [Expanding along C_1]

$= (1+x+x^2)(1-x)^2(1+x+x^2) = (1-x^3)^2 = \text{RHS}$

Question 13:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Answer 13:

$$\text{LHS} = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$= \frac{1}{a}\begin{vmatrix} 1+a^2-b^2 & 2ab & -2ab \\ 2ab & 1-a^2+b^2 & 2a^2 \\ 2b & -2a & a-a^3-ab^2 \end{vmatrix} \quad [\text{Applying } C_3 \to aC_3]$$

$$= \frac{1}{a}\begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2ab & 1-a^2+b^2 & 1+a^2+b^2 \\ 2b & -2a & -a-a^3-ab^2 \end{vmatrix} \quad [\text{Applying } C_3 \to C_3 + C_2]$$

$$= \frac{1+a^2+b^2}{a}\begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2ab & 1-a^2+b^2 & 1 \\ 2b & -2a & -a \end{vmatrix}$$

[Taking $1+a^2+b^2$ as common from C_3]

$$= \frac{1+a^2+b^2}{a^2}\begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2a^2b & a-a^3+ab^2 & a \\ 2b & -2a & -a \end{vmatrix} \quad [\text{Applying } R_2 \to aR_2]$$

$$= \frac{1+a^2+b^2}{a^2}\begin{vmatrix} 1+a^2-b^2 & 2ab & 0 \\ 2a^2b+2b & -a-a^3+ab^2 & 0 \\ 2b & -2a & -a \end{vmatrix} \quad [\text{Applying } R_2 \to R_2 + R_3]$$

$$= (1+a^2+b^2)\begin{vmatrix} 1+a^2-b^2 & 2b & 0 \\ 2a^2b+2b & -1-a^2+b^2 & 0 \\ 2b & -2 & -1 \end{vmatrix}$$

[Taking a as common from C_2 and C_3]

$= (1+a^2+b^2)(-1)\{(1+a^2-b^2)(-1-a^2+b^2) - 2b(2a^2b+2b)\}$

[Expanding along C_3]

$= -(1+a^2+b^2)[-1-a^2+b^2-a^2-a^4+a^2b^2+b^2+a^2b^2-b^4-4a^2b^2-4b^2]$

$= (1+a^2+b^2)\{1+a^4+4+2a^2+2a^2b^2+2b^2\}$

$= (1+a^2+b^2)(1+a^2+b^2)^2 = (1-x^3)^2 = \text{RHS}$

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Question 14:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

Answer 14:

$$\text{LHS} = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^3+a & a^2b & a^2c \\ ab^2 & b^3+b & b^2c \\ c^2a & c^2b & c^3+c \end{vmatrix} \quad [\text{Applying } R_1 \to aR_1, R_3 \to bR_3, R_3 \to cR_3]$$

$$= \frac{abc}{abc}\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

[Taking a as common from C_1, b from C_2 and c from C_3]

$$= \begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

[By $R_1 \to R_1 + R_2 + R_3$]

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

[Taking $1 + a^2 + b^2 + c^2$ as common from R_1]

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & b^2 \\ 0 & -1 & c^2+1 \end{vmatrix} \quad [\text{Applying } C_1 \to C_1 - C_2, C_2 \to C_2 - C_3]$$

$$= (1+a^2+b^2+c^2)\{1-0\} \quad [\text{Expanding along } R_1]$$

$$= 1 + a^2 + b^2 + c^2 = \text{RHS}$$

(Class 12)

Choose the correct answer in Exercises 15 and 16.

Question 15:

Let A be a square matrix of order 3×3, then $|kA|$ is equal to:

(A) $k|A|$ (B) $k^2|A|$ (C) $k^3|A|$ (D) $3k|A|$

Answer 15:

If B be a square matrix of order $n \times n$, then $|kB| = k^{n-1}|B|$

Therefore, $|kA| = k^{3-1}|A| = k^2|A|$

Hence, the option (B) is correct.

Question 16:

Which of the following is correct

(A) Determinant is a square matrix.

(B) Determinant is a number associated to a matrix.

(C) Determinant is a number associated to a square matrix.

(D) None of these

Answer 16:

Determinant is a number associated to a square matrix.

Hence, the option (C) is correct.

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Exercise 4.3

Question 1:

Find area of the triangle with vertices at the point given in each of the following:

(i) $(1,0),(6,0),(4,3)$

(ii) $(2,7),(1,1),(10,8)$

(iii) $(-2,-3),(3,2),(-1,-8)$

Answer 1:

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(i) $A(1,0), B(6,0), C(4,3)$

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$

$= \frac{1}{2}[1(0-3) - 0(6-4) + 1(18-0)] = \frac{1}{2}(15) = 7.5$ square units

(ii) $A(2,7),\ B(1,1),\ C(10,8)$

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$

$= \frac{1}{2}[2(1-8) - 7(1-10) + 1(8-10)] = \frac{1}{2}(47) = 25.5$ square units

(iii) $A(-2,-3), B(3,2), C(-1,-8)$

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$

$= \frac{1}{2}[-2(2+8) + 3(3+1) + 1(-24+2)] = \frac{1}{2}(-30) = -15$

Area of triangle $ABC = 15$ square units

Question 2:

Show that points $A(a, b+c), B(b, c+a), C(c, a+b)$ are collinear.

Answer 2:

If the points $A(a, b+c), B(b, c+a)$ and $C(c, a+b)$ are collinear, the area of triangle ABC will be zero.

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} a & a+b+c & 1 \\ b & a+b+c & 1 \\ c & a+b+c & 1 \end{vmatrix}$ [Applying $C_2 \to C_1 + C_2$]

$= \frac{1}{2}(a+b+c)\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix}$ [Taking $a+b+c$ as common from C_2]

$= 0$ [$\because C_1 = C_3$]

Hence, the points $A(a, b+c), B(b, c+a)$ and $C(c, a+b)$ are collinear.

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Question 3:

Find values of k if area of triangle is 4 sq. units and vertices are

(i) $(k, 0), (4, 0), (0, 2)$ **(ii)** $(-2, 0), (0, 4), (0, k)$

Answer 3:

(i) $A(k, 0), B(4, 0), C(0, 2)$

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$= \dfrac{1}{2}[k(0-2) - 0(4-0) + 1(8-0)] = \dfrac{1}{2}(-2k+8) = -k+4$

According to question, Area of triangle $ABC = 4$ square units

Therefore, $|-k+4| = 4 \quad \Rightarrow -k+4 = \pm 4$

$\Rightarrow -k+4 = 4 \quad$ or $\quad -k+4 = -4$

$\Rightarrow k = 0 \quad$ or $\quad k = 8$

Hence, the value of k are 0 and 8.

(ii) $A(-2, 0), B(0, 4), C(0, k)$

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$

$= \dfrac{1}{2}[-2(4-k) - 0(0-0) + 1(0-0)] = \dfrac{1}{2}(-8+2k) = -4+k$

According to question, Area of triangle $ABC = 4$ square units

Therefore, $|-4+k| = 4 \quad \Rightarrow -4+k = \pm 4$

$\Rightarrow -4+k = 4 \quad$ or $\quad -4+k = -4$

$\Rightarrow k = 8 \quad$ or $\quad k = 0$

Hence, the value of k are 0 and 8.

Question 4:

(i) Find equation of line joining (1, 2) and (3, 6) using determinants.

(ii) Find equation of line joining (3, 1) and (9, 3) using determinants.

Answer 4:

(i) Let, $P(x, y)$ be any point lie on the line joining $A(1, 2)$ and $B(3, 6)$. Hence, the points A, B and P will be collinear and area of triangle ABC will be zero.

Therefore, Area of triangle $ABP = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$

$\Rightarrow \dfrac{1}{2}[1(6-y) - 2(3-x) + 1(3y-6x)] = 0$

$\Rightarrow 6 - y - 6 + 2x + 3y - 6x = 0$

$\Rightarrow -4x + 2y = 0$

$\Rightarrow 2x = y$

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(ii) Let, $P(x, y)$ be any point lie on the line joining $A(3, 1)$ and $B(9, 3)$. Hence, the points A, B and P will be collinear and area of triangle ABC will be zero.

Therefore, Area of triangle $ABP = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$

$\Rightarrow \frac{1}{2}[3(3 - y) - 1(9 - x) + 1(9y - 3x)] = 0$
$\Rightarrow 9 - 3y - 9 + x + 9y - 3x = 0$
$\Rightarrow -2x + 6y = 0$
$\Rightarrow x = 3y$

Question 5:

If area of triangle is 35 sq. units with vertices (2, – 6), (5, 4) and (k, 4). Then k is
(A) 12 (B) -2 (C) $-12, -2$ (D) $12, -2$

Answer 5:

$A(2, -6), B(5, 4), C(k, 4)$

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$

$= \frac{1}{2}[2(4 - 4) + 6(5 - k) + 1(20 - 4k)] = \frac{1}{2}(30 - 6k + 20 - 4k) = 25 - 5k$

According to question, Area of triangle $ABC = 35$ square units

Therefore, $|25 - 5k| = 35$

$\Rightarrow 25 - 5k = \pm 35$

$\Rightarrow 25 - 5k = 35$ or $25 - 5k = -35$

$\Rightarrow k = \frac{-10}{5} = -2$ or $k = \frac{60}{5} = 12$

Hence, the option (D) is correct.

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Exercise 4.4

Write Minors and Cofactors of the elements of following determinants:

Question 1:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ **(ii)** $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Answer 1:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

The minor of element a_{ij} is M_{ij} and the cofactor is $A_{ij} = (-1)^{i+j}M_{ij}$, therefore,

The minor of element a_{11} is M_{11}= 3 and the cofactor is $A_{11} = (-1)^{1+1}M_{11} = 3$

The minor of element a_{12} is M_{12}= 0 and the cofactor is $A_{12} = (-1)^{1+2}M_{12} = 0$

The minor of element a_{21} is M_{21}= -4 and the cofactor is $A_{21} = (-1)^{2+1}M_{21} = 4$

The minor of element a_{22} is M_{22}= 2 and the cofactor is $A_{22} = (-1)^{2+2}M_{22} = 2$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

The minor of element a_{11} is M_{11}= d and the cofactor is $A_{11} = (-1)^{1+1}M_{11} = d$

The minor of element a_{12} is M_{12}= b and the cofactor is $A_{12} = (-1)^{1+2}M_{12} = -b$

The minor of element a_{21} is M_{21}= c and the cofactor is $A_{21} = (-1)^{2+1}M_{21} = -c$

The minor of element a_{22} is M_{22}= a and the cofactor is $A_{22} = (-1)^{2+2}M_{22} = a$

Question 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ **(ii)** $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Answer 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$,

Here,

$$M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1, \quad M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0, \quad M_{13} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$$

$$M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0, \quad M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1, \quad M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$M_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0, \quad M_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0, \quad M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

and $A_{ij} = (-1)^{i+j}M_{ij}$, therefore

$A_{11} = (-1)^{1+1}M_{11} = 1$ $A_{12} = (-1)^{1+2}M_{12} = 0$ $A_{13} = (-1)^{1+3}M_{13} = 0$

$A_{21} = (-1)^{2+1}M_{21} = 0$ $A_{22} = (-1)^{2+2}M_{22} = 1$ $A_{23} = (-1)^{2+3}M_{233} = 0$

$A_{31} = (-1)^{3+1}M_{31} = 0$ $A_{32} = (-1)^{3+2}M_{32} = 0$ $A_{33} = (-1)^{3+3}M_{33} = 1$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$,

Here,

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$M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11,\ M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6,\ M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$

$M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4,\quad M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2,\quad M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

$M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20, M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13\ M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$

and $A_{ij} = (-1)^{i+j}M_{ij}$, therefore

$A_{11} = (-1)^{1+1}M_{11} = 11 \qquad A_{12} = (-1)^{1+2}M_{12} = -6 \qquad A_{13} = (-1)^{1+3}M_{13} = 3$

$A_{21} = (-1)^{2+1}M_{21} = 4 \qquad A_{22} = (-1)^{2+2}M_{22} = 2 \qquad A_{23} = (-1)^{2+3}M_{233} = -1$

$A_{31} = (-1)^{3+1}M_{31} = -20 \qquad A_{32} = (-1)^{3+2}M_{32} = 13 \qquad A_{33} = (-1)^{3+3}M_{33} = 5$

Question 3:

Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

Answer 3:

$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$

Here, $a_{21} = 2,\ a_{22} = 0,\ a_{23} = 1$ and

$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = -(9 - 16) = 7$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10 - 3) = -7$

Therefore, $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 2(7) + 0(7) + 1(-7) = 7$

Question 4:

Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

Answer 4:

$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

Here, $a_{13} = yz,\ a_{23} = zx,\ a_{33} = xy$ and

$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = -(z - x) = x - z$

$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$

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Therefore, $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = yz(z-y) + zx(x-z) + xy(y-x)$

$= yz^2 - y^2z + zx^2 - xz^2 + xy^2 - x^2y$
$= zx^2 - x^2y - xz^2 + xy^2 + yz^2 - y^2z$
$= x^2(z-y) - x(z^2-y^2) + yz(z-y)$
$= (z-y)[x^2 - x(z+y) + yz]$
$= (z-y)[x^2 - xz - xy + yz]$
$= (z-y)[x(x-z) - y(x-z)]$
$= (x-z)(z-y)(x-y)$
$= (x-y)(y-z)(z-x)$

Question 5:

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ A_{ij} is Cofactors of a_{ij}, then value of Δ is given by

(A) $a_{11}A_{11} + a_{12}A_{32} + a_{13}A_{33}$ (B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ (D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Answer 5:

The value of $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ is given by: $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Hence, the option (D) is correct.

Exercise 4.5

Find adjoint of each of the matrices in Exercises 1 and 2.

Question 1:

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Answer 1:

Here, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, therefore, $A_{11} = 4$ $A_{12} = -3$ $A_{21} = -2$ $A_{22} = 1$

Adjoint of matrix $A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Question 2:

$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

Answer 2:

Here, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$, therefore

$A_{11} = 3$ $A_{12} = -12$ $A_{13} = 6$
$A_{21} = 1$ $A_{22} = 5$ $A_{23} = 2$
$A_{31} = -11$ $A_{32} = -1$ $A_{33} = 5$

Adjoint of matrix $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$

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Verify $A(adj\ A) = (adj\ A).A = |A|.I$ in Exercises 3 and 4

Question 3:

$$\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

Answer 3:

Here, $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$, therefore,

$A_{11} = -6 \quad A_{12} = 4 \quad A_{21} = -3 \qquad A_{22} = 2$

$|A| = -12 + 12 = 0$

$$adj\ A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$A(adj\ A) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$(adj\ A).A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} = \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$|A|.I = 0.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $A(adj\ A) = (adj\ A).A = |A|.I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Question 4:

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

Answer 4:

Here, $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, therefore, $|A| = 1(0-0) + 1(9+2) + 2(0-0) = 11$

$A_{11} = 0 \qquad A_{12} = -11 \qquad A_{13} = 0$

$A_{21} = 3 \qquad A_{22} = 1 \qquad A_{23} = -1$

$A_{31} = 2 \qquad A_{32} = 8 \qquad A_{33} = 3$

Adjoint of matrix $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$

$$A(adj\ A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0+0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$(adj\ A).A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

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$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0+0+0 & 0+2+9 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$|A|.I = 11.\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Hence, $A(adj\ A) = (adj\ A).A = |A|.I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

Find the inverse of each of the matrices (if it exists) given in Exercises 5 to 11.

Question 5:

$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

Answer 5:

Here, $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$,

Therefore, $A_{11} = 3 \quad A_{12} = -4 \quad A_{21} = 2 \qquad A_{22} = 2$

$|A| = 6 + 8 = 14 \neq 0 \quad \Rightarrow A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

Question 6:

$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

Answer 6:

Here, $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$,

Therefore, $A_{11} = 2 \quad A_{12} = 3 \quad A_{21} = -5 \qquad A_{22} = -1$

$|A| = -2 + 15 = 13 \neq 0 \quad \Rightarrow A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

Question 7:

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Answer 7:

Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$,

Therefore, $|A| = 1(10-0) - 2(0-0) + 3(0-0) = 10 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = 10 \qquad A_{12} = 0 \qquad A_{13} = 0$

$A_{21} = -10 \qquad A_{22} = 5 \qquad A_{23} = 0$

$A_{31} = 2 \qquad A_{32} = -4 \qquad A_{33} = 2$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

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Question 8:

$$\begin{bmatrix}1 & 0 & 0\\ 3 & 3 & 0\\ 5 & 2 & -1\end{bmatrix}$$

Answer 8:

Here, $A = \begin{bmatrix}1 & 0 & 0\\ 3 & 3 & 0\\ 5 & 2 & -1\end{bmatrix}$,

Therefore, $|A| = 1(-3-0) - 0(-3-0) + 0(6-15) = -3 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = -3$	$A_{12} = 3$	$A_{13} = -9$
$A_{21} = 0$	$A_{22} = -1$	$A_{23} = -2$
$A_{31} = 0$	$A_{32} = 0$	$A_{33} = 3$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix} = \frac{1}{-3}\begin{bmatrix}-3 & 0 & 0\\ 3 & -1 & 0\\ -9 & -2 & 3\end{bmatrix}$$

Question 9:

$$\begin{bmatrix}2 & 1 & 3\\ 4 & -1 & 0\\ -7 & 2 & 1\end{bmatrix}$$

Here, $A = \begin{bmatrix}2 & 1 & 3\\ 4 & -1 & 0\\ -7 & 2 & 1\end{bmatrix}$,

Therefore, $|A| = 2(-1-0) - 1(4-0) + 3(8-7) = -3 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = -1$	$A_{12} = -4$	$A_{13} = 1$
$A_{21} = 5$	$A_{22} = 23$	$A_{23} = -11$
$A_{31} = 3$	$A_{32} = 12$	$A_{33} = -6$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix} = \frac{1}{-3}\begin{bmatrix}-1 & 5 & 3\\ -4 & 23 & 12\\ 1 & -11 & -6\end{bmatrix}$$

Question 10:

$$\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$$

Answer 10:

Here, $A = \begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$,

Therefore, $|A| = 1(8-6) + 1(0+9) + 2(0-6) = -1 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = 2$	$A_{12} = -9$	$A_{13} = -6$
$A_{21} = 0$	$A_{22} = -2$	$A_{23} = -1$
$A_{31} = -1$	$A_{32} = 3$	$A_{33} = 2$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix} = \frac{1}{-1}\begin{bmatrix}2 & 0 & -1\\ -9 & -2 & 3\\ -6 & -1 & 2\end{bmatrix} = \begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$$

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Question 11:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

Answer 11:

Here, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$, therefore

$|A| = 1(-\cos^2\alpha - \sin^2\alpha) + 0(0-0) + 0(0-0) = -1 \neq 0$

$\Rightarrow A^{-1}$ exists.

$A_{11} = 1$ $\quad A_{12} = 0$ $\quad A_{13} = 0$

$A_{21} = 0$ $\quad A_{22} = -\cos\alpha$ $\quad A_{23} = -\sin\alpha$

$A_{31} = 0$ $\quad A_{32} = -\sin\alpha$ $\quad A_{33} = \cos\alpha$

$$A^{-1} = \frac{1}{|A|} adj\, A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

Question 12.

Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Answer 12:

Here, $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$, therefore, $A_{11} = 5 \quad A_{12} = -2 \quad A_{21} = -7 \quad A_{22} = 3$

$|A| = 15 - 14 = 1 \neq 0 \quad \Rightarrow A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|} adj\, A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$, therefore, $B_{11} = 9 \quad B_{12} = -7 \quad B_{21} = -8 \quad B_{22} = 6$

$|A| = 54 - 56 = -2 \neq 0 \quad \Rightarrow B^{-1}$ exists.

$$B^{-1} = \frac{1}{|B|} adj\, B = \frac{1}{|B|}\begin{bmatrix} B_{11} & B_{21} \\ B_{12} & B_{22} \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -9/2 & 4 \\ 7/2 & -3 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} -9/2 & 4 \\ 7/2 & -3 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -\frac{45}{2} - 8 & \frac{63}{2} + 12 \\ \frac{35}{2} + 6 & -\frac{49}{2} - 9 \end{bmatrix} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix}$$

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$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

$|AB| = 67 \times 61 - 87 \times 47 = 4087 - 4089 = -2 \neq 0 \quad \Rightarrow (AB)^{-1}$ exists.

$C_{11} = 61 \quad C_{12} = -47 \quad C_{21} = -87 \quad C_{22} = 67$

$$(AB)^{-1} = \frac{1}{|AB|} adj \text{ AB} = \frac{1}{|AB|}\begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix}$$

Hence, $(AB)^{-1} = B^{-1}A^{-1}$ is verified.

Question 13:

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = O$. Hence, find A^{-1}.

Answer 13:

LHS $= A^2 - 5A + 7I = AA - 5A + 7I$

$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 =$ RHS

$\Rightarrow A^2 - 5A + 7I = O$

$\Rightarrow A^2 - 5A = -7I$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$AAA^{-1} - 5AA^{-1} = -7IA^{-1}$

$\Rightarrow AI - 5I = -7A^{-1}$ [Because $AA^{-1} = I$]

$\Rightarrow 7A^{-1} = 5I - A = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Question 14:

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For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$.

Answer 14:

Given that: $A^2 + aA + bI = O$

$\Rightarrow \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow 4 + a = 0 \quad \Rightarrow a = -4 \quad$ and $\quad 3 + a + b = 0 \ \Rightarrow b = -3 - a = -3 + 4 = 1$

Hence, $a = -4, \ b = 1$

Question 15:

For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11\,I = O$. Hence, find A^{-1}.

Answer 15:

$A^2 = A.A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$

$A^3 = A^2.A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$

$\text{LHS} = A^3 - 6A^2 + 5A + 11\,I$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

$= \begin{bmatrix} 8-24+5+11 & 7-12+5+0 & 1-6+5+0 \\ -23+18+5+0 & 27-48+10+11 & -69+84-15+0 \\ 32-42+10+0 & -13+18-5+0 & 58-84+15+11 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O = \text{RHS}$

$\Rightarrow A^3 - 6A^2 + 5A + 11\,I = O \qquad \Rightarrow A^3 - 6A^2 + 5A = -11I$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$A^2AA^{-1} - 6AAA^{-1} + 5AA^{-1} = -11IA^{-1}$

$\Rightarrow A^2I - 6AI + 5I = -11A^{-1} \qquad$ [Because $AA^{-1} = I$]

$\Rightarrow 11A^{-1} = -A^2 + 6A - 5I$

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$$\Rightarrow 11A^{-1} = -\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix} + \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} -4+6-5 & -2+6+0 & -1+6+0 \\ 3+6-0 & -8+12-5 & 14-18+0 \\ -7+12+0 & 3-6+0 & -14+18-5 \end{bmatrix} = \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

Question 16:

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}.

Answer 16:

$$A^2 = A.A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$\text{LHS} = A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36+18-4 & -21+30-9+0 & 21-30+9+0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9+0 \\ 21-30+9-0 & -21+30-9-0 & 22-36+18-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{RHS}$$

$\Rightarrow A^3 - 6A^2 + 9A - 4I = O \qquad \Rightarrow A^3 - 6A^2 + 9A = 4I$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$A^2AA^{-1} - 6AAA^{-1} + 9AA^{-1} = 4IA^{-1}$

$\Rightarrow A^2I - 6AI + 9I = 4A^{-1}$ [Because $AA^{-1} = I$]

$\Rightarrow 4A^{-1} = A^2 - 6A + 9I$

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$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Question 17:

Let A be a non-singular square matrix of order 3 × 3. Then $|adj\ A|$ is equal to:

(A) $|A|$ (B) $|A|^2$ (C) $|A|^3$ (D) $3|A|$

Answer 17:

We know that $adj\ A = |A|I$

$$\Rightarrow (adj\ A)A = |A|\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |(adj\ A)A| = |A|\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix} = |A|^3$$

$\Rightarrow |adj\ A| = |A|^2$,

Hence, the option (B) is correct.

Question 18:

If A is an invertible matrix of order 2, then $det(A^{-1})$ is equal to:

(A) $\det(A)$ (B) $\frac{1}{\det(A)}$ (C) 1 (D) 0

Answer 18:

Given that the matrix A is invertible, hence, $A^{-1} = \frac{1}{|A|} adj\ A$

The order of matrix is 2, so, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Therefore, $|A| = ad - bc$ तथा $adj\ A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} adj\ A = A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} \frac{d}{|A|} & -\frac{b}{|A|} \\ -\frac{c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$det(A^{-1}) = |A^{-1}| = \begin{vmatrix} \frac{d}{|A|} & -\frac{b}{|A|} \\ -\frac{c}{|A|} & \frac{a}{|A|} \end{vmatrix}$$

$$= \frac{1}{|A|^2}\begin{vmatrix} d & -b \\ -c & a \end{vmatrix} = \frac{1}{|A|^2}(ad - bc) = \frac{1}{|A|^2}|A| = \frac{1}{|A|}$$

Hence, the option (B) is correct.

Mathematics

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(Chapter – 4) (Determinants)
(Class 12)
Exercise 4.6

Examine the consistency of the system of equations in Exercises 1 to 6.

Question 1:

$x + 2y = 2$

$2x + 3y = 3$

Answer 1:

The given system of equations: $\begin{aligned} x + 2y &= 2 \\ 2x + 3y &= 3 \end{aligned}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$|A| = 3 - 4 = -1 \neq 0 \quad \Rightarrow$ A is non-singular and so A^{-1} exists.

Hence, the system of equations are consistent.

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Question 2:

$2x - y = 5$
$x + y = 4$

Answer 2:

The given system of equations: $\begin{matrix} 2x - y = 5 \\ x + y = 4 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$|A| = 2 + 1 = 3 \neq 0 \quad \Rightarrow A$ is non-singular and so A^{-1} exists.
Hence, the system of equations are consistent.

Question 3:

$x + 3y = 5$
$2x + 6y = 8$

Answer 3:

The given system of equations: $\begin{matrix} x + 3y = 5 \\ 2x + 6y = 8 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$|A| = 6 - 6 = 0 \quad \Rightarrow A$ is a singular matrix and so A^{-1} does not exists. Now,

$adj\ A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

$(adj\ A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq O$

So, there is no solutions of the given system of equations.
Hence, the system of equations are inconsistent.

Question 4:

$x + y + z = 1$
$2x + 3y + 2z = 2$
$ax + ay + 2az = 4$

Answer 4:

The given system of equations: $\begin{matrix} x + y + z = 1 \\ 2x + 3y + 2z = 2 \\ ax + ay + 2az = 4 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$|A| = 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a) = a \neq 0$
$\Rightarrow A$ is non-singular and so A^{-1} exists. Now,
Hence, the system of equations are consistent.

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Question 5:

$3x - y - 2z = 2$
$2y - z = -1$
$3x - 5y = 3$

Answer 5:

The given system of equations:
$$\begin{aligned} 3x - y - 2z &= 2 \\ 2y - z &= -1 \\ 3x - 5y &= 3 \end{aligned}$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$|A| = 3(0 - 5) + 1(0 + 3) - 2(0 - 6) = -15 + 3 + 12 = 0$

$\Rightarrow$ A is a singular matrix and so A^{-1} does not exists. Now,

$A_{11} = -5$ $\quad A_{12} = -3$ $\quad A_{13} = -6$

$A_{21} = 10$ $\quad A_{22} = 6$ $\quad A_{23} = 12$

$A_{31} = 5$ $\quad A_{32} = 3$ $\quad A_{33} = 6$

$$adj\ A = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$(adj\ A)B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 - 10 + 15 \\ -6 - 6 + 9 \\ -12 - 12 + 16 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq O$$

So, there is no solutions of the given system of equations.
Hence, the system of equations are inconsistent.

Question 6:

$5x - y + 4z = 5$
$2x + 3y + 5z = 2$
$5x - 2y + 6z = -1$

Answer 6:

The given system of equations:
$$\begin{aligned} 5x - y + 4z &= 5 \\ 2x + 3y + 5z &= 2 \\ 5x - 2y + 6z &= -1 \end{aligned}$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$|A| = 5(18 + 10) + 1(12 - 25) + 4(-4 - 15) = 140 - 13 - 76 = 51 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Hence, the system of equations are consistent.

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Solve system of linear equations, using matrix method, in Exercises 7 to 14.

Question 7:

$5x + 2y = 4$
$7x + 3y = 5$

Answer 7:

The given system of equations: $\begin{matrix} 5x + 2y = 4 \\ 7x + 3y = 5 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$|A| = 15 - 14 = 1 \neq 0 \quad \Rightarrow A$ is non-singular and so A^{-1} exists.
Hence, the system of equations are consistent.
Now, $A_{11} = 3 \quad A_{12} = -7 \quad A_{21} = -2 \quad A_{22} = 5$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \quad \Rightarrow x = 2, \quad y = -3$$

Question 8:

$2x - y = -2$
$3x + 4y = 3$

Answer 8:

The given system of equations: $\begin{matrix} 2x - y = -2 \\ 3x + 4y = 3 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$|A| = 8 + 3 = 11 \neq 0 \quad \Rightarrow A$ is non-singular and so A^{-1} exists. Now,
Hence, the system of equations are consistent.
Now, $A_{11} = 4 \quad A_{12} = -3 \quad A_{21} = 1 \quad A_{22} = 2$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -8 + 3 \\ 6 + 6 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix} \quad \Rightarrow x = -\frac{5}{11}, \quad y = \frac{12}{11}$$

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Question 9:
$4x - 3y = 3$
$3x - 5y = 7$

Answer 9:

The given system of equations: $\begin{matrix} 4x - 3y = 3 \\ 3x - 5y = 7 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$|A| = -20 + 9 = -11 \neq 0 \quad \Rightarrow A$ is non-singular and so A^{-1} exists.
Hence, the system of equations are consistent.
Now, $A_{11} = -5 \quad A_{12} = -3 \quad A_{21} = 3 \quad A_{22} = 4$

$$A^{-1} = \frac{1}{|A|} adj\, A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11}\begin{bmatrix} -15 + 21 \\ -9 + 28 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{6}{11} \\ -\frac{19}{11} \end{bmatrix} \quad \Rightarrow x = -\frac{6}{11}, \quad y = -\frac{19}{11}$$

Question 10:
$5x + 2y = 3$
$3x + 2y = 5$

Answer 10:

The given system of equations: $\begin{matrix} 5x + 2y = 3 \\ 3x + 2y = 5 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$|A| = 10 - 6 = 4 \neq 0 \quad \Rightarrow A$ is non-singular and so A^{-1} exists.
Hence, the system of equations are consistent.
Now, $A_{11} = 2 \quad A_{12} = -3 \quad A_{21} = -2 \quad A_{22} = 5$

$$A^{-1} = \frac{1}{|A|} adj\, A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6 - 10 \\ -9 + 25 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{4}{4} \\ \frac{16}{4} \end{bmatrix} \quad \Rightarrow x = -1, \quad y = 4$$

Question 11:
$2x + y + z = 1$
$x - 2y - z = \frac{3}{2}$
$3y - 5z = 9$

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Answer 11:

The given system of equations:
$$2x + y + z = 1$$
$$x - 2y - z = \frac{3}{2}$$
$$3y - 5z = 9$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

$|A| = 2(10 + 3) - 1(-5 - 0) + 1(3 - 0) = 26 + 5 + 3 = 34 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 13$ $\quad A_{12} = 5$ $\quad A_{13} = 3$

$A_{21} = 8$ $\quad A_{22} = -10$ $\quad A_{23} = -6$

$A_{31} = 1$ $\quad A_{32} = 3$ $\quad A_{33} = -5$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 + 12 + 9 \\ 5 - 15 + 27 \\ 3 - 9 - 45 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -3/2 \end{bmatrix} \Rightarrow x = 1, y = \frac{1}{2}, z = -\frac{3}{2}$$

QUESTIONS 12,13 AND 14 TRY YOURSELF

Question 15:

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Answer 15:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$|A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = 0 - 6 + 5 = -1 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 0$ $\quad A_{12} = 2$ $\quad A_{13} = 1$

$A_{21} = -1$ $\quad A_{22} = -9$ $\quad A_{23} = -5$

$A_{31} = 2$ $\quad A_{32} = 23$ $\quad A_{33} = 13$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations:
$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

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This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 1, y = 2, z = 3$

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Question 16:

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹ 70. Find cost of each item per kg by matrix method.

Answer 16:

Let the cost of 1 kg of onion = ₹ x,
Let the cost of 1 kg of wheat = ₹ y and
Let the cost of 1 kg rice = ₹ z
The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. So $4x + 3y + 2z = 60$
The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. So, $2x + 4y + 6z = 90$ and
The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹ 70. So, $6x + 2y + 3z = 70$

The given system of equations: $\begin{matrix} 4x + 3y + 2z = 60 \\ 2x + 4y + 6z = 90 \\ 6x + 2y + 3z = 70 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$|A| = 4(12 - 12) - 3(6 - 36) + 2(4 - 24) = 0 + 90 - 40 = 50 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 0$ $\quad A_{12} = 30$ $\quad A_{13} = -20$
$A_{21} = -5$ $\quad A_{22} = 0$ $\quad A_{23} = 10$
$A_{31} = 10$ $\quad A_{32} = -20$ $\quad A_{33} = 10$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\Rightarrow x = 5, y = 8, z = 8$

Hence, the cost of 1 kg of onion is ₹ 7, the cost of 1 kg of wheat is ₹ 8 and the cost of 1 ka rice is ₹ 8.

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Question 16:

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹ 70. Find cost of each item per kg by matrix method.

Answer 16:

Let the cost of 1 kg of onion = ₹ x,
Let the cost of 1 kg of wheat = ₹ y and
Let the cost of 1 kg rice = ₹ z
The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. So $4x + 3y + 2z = 60$
The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. So, $2x + 4y + 6z = 90$ and
The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹ 70. So, $6x + 2y + 3z = 70$

The given system of equations: $\begin{aligned} 4x + 3y + 2z &= 60 \\ 2x + 4y + 6z &= 90 \\ 6x + 2y + 3z &= 70 \end{aligned}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$|A| = 4(12 - 12) - 3(6 - 36) + 2(4 - 24) = 0 + 90 - 40 = 50 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 0$ $\quad A_{12} = 30$ $\quad A_{13} = -20$
$A_{21} = -5$ $\quad A_{22} = 0$ $\quad A_{23} = 10$
$A_{31} = 10$ $\quad A_{32} = -20$ $\quad A_{33} = 10$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\Rightarrow x = 5, y = 8, z = 8$

Hence, the cost of 1 kg of onion is ₹ 7, the cost of 1 kg of wheat is ₹ 8 and the cost of 1 ka rice is ₹ 8.

ALL WORK IS TO BE DONE IN
MATHS CLASSWORK REGISTER
IT WILL BE CHECKED WHEN SCHOOL RE-OPENS.

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SELF EVALUATION TEST (10 MARKS)

If you have the belief that you can do it, you will acquire all the capacity to do it even if you may not have it at the beginning! – AKS

1.

If $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ and, $B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$ are two square matrices, find AB.

Hence solve the following system of linear equations :

$x-y=3,$

$2x+3y+4z=17$ and,

$y+2z=7.$

2.

Prove that $\begin{vmatrix}yz-x^2 & zx-y^2 & xy-z^2\\ zx-y^2 & xy-z^2 & yz-x^2\\ xy-z^2 & yz-x^2 & zx-y^2\end{vmatrix}$ is divisible by $(x+y+z)$, and hence find the quotient.

3.

Using properties of determinants, prove that $\begin{vmatrix}(y+z)^2 & xy & zx\\ xy & (x+z)^2 & yz\\ xz & yz & (x+y)^2\end{vmatrix}=2xyz(x+y+z)^3$.

4.

Using elementary transformations, find the inverse of the matrix : $\begin{pmatrix}1 & 3 & -2\\ -3 & 0 & -1\\ 2 & 1 & 0\end{pmatrix}$.

5.

If $A=\begin{bmatrix}2 & 3 & 4\\ 1 & -1 & 0\\ 0 & 1 & 2\end{bmatrix}$, find A^{-1}.

Hence solve the following system of linear equations :

$x-y=3,$

$2x+3y+4z=17$ and,

$y+2z=7.$

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OR

If a, b, c are p^{th}, q^{th} and r^{th} terms respectively of a G.P., then prove that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$.

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Compiled by: AKS (PGT: MATHS)
ST. MARY'S PUBLIC SCHOOL

THANKS

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